

1. A ferris wheel has a diameter of 24 metres and rotates once each 46 seconds. If the bottom of the ferris wheel is raised 2 metres above the ground and the ride starts from the bottom of the ferris wheel:

- (a) Write an equation to relate the height,  $y$  metres above the ground, of a person  $t$  seconds after the ride starts.

$$y = 12 \cos \frac{2\pi}{46} (t - 23) + 14$$

$$y = -12 \cos \frac{2\pi}{46} t + 14$$

- (b) Find the time it takes for the height to reach 20 metres for the first time (to the nearest tenth of a second).

$$20 = 12 \cos \frac{2\pi}{46} (t - 23) + 14$$

$$t = \frac{46}{2\pi} \cos^{-1} \left( \frac{20 - 14}{12} \right) + 23$$

$$t = 30.7 \text{ sec.}$$

$$\text{First time: } 46 - 30.7 = 15.3 \text{ sec.}$$

2. A high tide depth of 7.6 m occurs at 5:25 AM and low tide depth of 1.2 m occurs at 11:45 AM.

- (a) Write an equation to relate the depth,  $y$  metres, of the water  $t$  hours after midnight.

$$\text{Amplitude} = \frac{7.6 - 1.2}{2} = 3.2$$

$$\text{Vertical Displacement} = 7.6 - 3.2 = 4.4$$

$$\text{Phase shift: } 5:25 \text{ AM} \rightarrow 5 \frac{25}{60} = 5 \frac{5}{12}$$

$$\text{Period: } (11 \frac{3}{4} - 5 \frac{5}{12}) \times 2 = 12 \frac{2}{3} = \frac{38}{3}$$

$$y = 3.2 \cos \frac{2\pi}{38/3} (t - 5 \frac{5}{12}) + 4.4 \text{ OR}$$

$$y = 3.2 \cos \frac{6\pi}{38} (t - 5 \frac{5}{12}) + 4.4 \text{ OR } y = 3.2 \cos \frac{3\pi}{19} (t - 5 \frac{5}{12}) + 4.4$$

- (b) Find the depth of the water at 3:40 pm (to the nearest tenth of a metre).

$$3:40 \text{ pm} \rightarrow 15 \frac{40}{60} = 15 \frac{2}{3}$$

$$y = 3.2 \cos \frac{3\pi}{19} (15 \frac{2}{3} - 5 \frac{5}{12}) + 4.4 = 5.6 \text{ m}$$

- (c) What time will the first high tide occur on the following day?

$$\text{First high tide on Day 1} = 5:25 \text{ AM}$$

$$= 5 \frac{5}{12} + 12 \frac{2}{3} = 18 \frac{1}{12} + 12 \frac{2}{3} = 30 \frac{3}{4}$$

$$30 \frac{3}{4} - 24 = 6 \frac{3}{4} = 6:45 \text{ AM}$$

3. The pedals of a bicycle have a maximum height of 32 cm and a minimum height of 10 cm above the ground. A person pedals at a constant rate of 20 cycles/minute. Find an equation to describe how the height,  $y$  metres above the ground, varies with the time,  $t$  seconds after starting if the pedal starts 21 cm above the ground and rising.

$$\text{period} = 20 \text{ cycles/min} \times \frac{1 \text{ min}}{60 \text{ sec}} = \frac{20}{60} = \frac{1}{3} \text{ cycles/sec} \leftrightarrow 3 \text{ sec/cycle}$$

$$y = 11 \sin \frac{2\pi}{3} t + 21 \text{ OR } y = 11 \cos \frac{2\pi}{3} (t - \frac{3}{4}) + 21$$

