BASIC RULES OF DERIVATIVES

1. Find the derivatives of the given functions:

a) $f(x)=x$ **b)** $f(x)=3x$ **c)** $f(x)=2x-1$ **d)** $f(x)=-x^2$

e) $f(x)=1-x^2$ **f)** $f(x)=2x^2+3$ **g)** $y=2x^2+3x-1$ **h)** $y=-\frac{5}{2}x^{40}+\frac{1}{2}x^{30}-\frac{5}{4}x^{20}$

i) $y=\sqrt{3}x^3+\sqrt{2}x^2-x-2$ **j)** $y=3x^4-2x^3+x+1$

2. Find the derivatives of the functions at the given points:

a) $f(x)=5x+1$ at $x=0$ and $x=2$

b) $f(x)=2x^2+3x+1$ at $x=-1$ and $x=1$

c) $f(x)=x^3+2x^2+3x+1$ at $x=-1$ and $x=2$

3. Find the derivatives of the given functions:

a) $y=(x-1)(x+1)$ **b)** $y=(2x+3)(3x+2)$ **c)** $y=(2x^3-1)(3x^2+1)$

d) $y=(x-2)(x+2)(x^2+4)$ **e)** $y=(x-1)(x^2+x+1)$ **f)** $y=(2x+3)(4x^2-6x+9)$

g) $f(x)=\dfrac{1}{x+1}$ **h)** $f(x)=\dfrac{-2}{x^2+1}$ **i)** $f(x)=\dfrac{x+1}{x-1}$

j) $f(x)=\dfrac{x^2+2}{2x^2+1}$ **k)** $f(x)=\dfrac{x^2+x+1}{x^2-x+1}$ **l)** $y=-\frac{1}{2}x+\frac{1}{2}x^{-4}+\frac{3}{2}x^{-8}$

m) $y=-5x^{10}+\dfrac{1}{x}$ **n)** $y=x-x^{-1}$ **o)** $y=x^{-2}+x^{-1}$

p) $y=(2x-1)(2x+1)(3x-2)(3x+2)$ **r)** $y=(x-1)(x^{99}+x^{98}+x^{97}+\ldots+1)$

s) $y=(-3x^2+1)(1-2x^3)(5x^2-x+1)$ **t)** $y=\dfrac{(x^2-4)(x^2+4)}{x^4}$

u) $y=\dfrac{(2x+1)(3x+1)}{x^2+1}$

4. Find the equations of the lines that are tangent and normal to the given functions at the given points:

a) $y=x(x+1)$ at $x=3$ **b)** $y=x^3-x+1$ at $x=2$

c) $y=(2x-1)(3x+1)$ at $x=0$ **d)** $y=\dfrac{x+1}{x-1}$ at $x=3$

e) $y=x+\dfrac{1}{x}+\dfrac{1}{x^2}$ at $x=-1$ **f)** $y=\dfrac{1}{x-1}+\dfrac{1}{x+1}$ at $x=-2$

5. Find $\tan\theta$ if θ is the angle between the x-axis and the line that is tangent to $y=\dfrac{x-1}{x+1}$ at the point $P\left(a,\dfrac{1}{2}\right)$.

6. Determine the points at which the tangents of the curve $y=\dfrac{x^3}{3}+\dfrac{3}{2}x^2$ are parallel to x-axis.

7. Let f be a function defined by $f: IR\to IR$, $f(x)=(m-1)x^2+3x-4$. Find m, if the tangent of f at $x=3$ makes the angle 135^0 with x-axis.

Basic Rules of Derivatives Worksheet

(1) a) 1 (b) 3 (c) 2 (d) $-2x$ (e) $-2x$

f) $4x$ (g) $4x+3$ (h) $40x^{-5/2}x^{39}+30\cdot\frac{1}{2}x^{29}-20\cdot\frac{5}{4}x^{19}$

$-100x^{39}+15x^{29}-25x^{19}$

i) $3\sqrt{3}x^{2}+2\sqrt{2}x-1$ j) $2x^{3}-8x^{2}+1$

(2) (a) 5 at both points $x=0$ and $x=2$

(b) $4x+3$ @ $x=-1$ $f'(-1)=-1$, @ $x=1$, $f'(1)=7$

(c) $3x^2+4x+3$ @ $x=-1$ $f'(-1)=2$, @ $x=2$, $f'(2)=23$

(3) $y=(x-1)(x+1)$

$=1(x+1)+1(x-1)$

$=x+1+x-1$

$=2x$

(b) $y=(2x+3)(3x+2)$

$=2(3x+2)+3(2x+3)$

$=6x+4+6x+9$

$=12x+13$

(c) $y=(2x^3-1)(3x^2+1)$

$=6x^2(3x^2+1)+6x(2x^3-1)$

$=18x^4+6x^2+12x^4-6x$

$=30x^4+6x^2-6x$

(d) $y=(x-2)(x+2)(x^2+4)$

$=(x^2-4)(x^2+4)$

$=2x(x^2+4)+2x(x^2-4)$

$=2x^3+8x+2x^3-8x$

$=4x^3$

(e) $(x-1)(x^2+x+1)$

$=x^2+x+1+(2x+1)(x-1)$

$=x^2+x+1+2x^2-2x+x-1$

$=3x^2$

(f) $(2x+3)(4x^2-6x+9)$

$=2(4x^2-6x+9)+(2x+3)(8x-6)$

$=8x^2-12x+18+16x^2-12x+24x-18$

$=24x^2$

(g) $(x+1)^{-1}=-(x+1)^{-2}$

$=\frac{-1}{x+1}$

(h) $\frac{-2}{x^2+1}\ \frac{u}{v}=\frac{vu'-uv'}{v^2}$

$=\frac{(x^2+1)(0)-(-2)(2x)}{(x^2+1)^2}$

$=\frac{4x}{(x^2+1)^2}$

(i) $\frac{x+1}{x-1}=\frac{vu'-uv'}{v^2}$

$\frac{(x-1)-(x+1)}{(x-1)^2}$

$=\frac{-2}{(x-1)^2}$

(j) $\frac{x^2+2}{2x^2+1}=\frac{vu'-uv'}{v^2}$

$=\frac{(2x^2+1)(2x)-(x^2+2)(4x)}{(2x^2+1)^2}$

$=\frac{4x^3+2x-4x^3-8x}{(2x^2+1)^2}$

$\frac{-6x}{(2x^2+1)^2}$

(k) $\frac{x^2+x+1}{x^2-x+1}=\frac{(x^2-x+1)(2x+1)-(x^2+x+1)(2x-1)}{(x^2-x+1)^2}$

$=\frac{2x^3+x^2-2x^2-x+2x+1-2x^3+x^2-2x^2+x-2x+1}{(x^2-x+1)^2}$

$=\frac{-2x^2-2}{(x^2-x+1)^2}$

l) $-\frac{1}{2}-2x^{-5}-12x^{-9}$ OR $-\frac{1}{2}-\frac{2}{x^5}-\frac{12}{x^9}$

(m) $-50x^{9}-x^{-2}$ OR $-50x^{9}-\frac{1}{x^2}$

(n) $1+x^{-2}$ OR $1+\frac{1}{x^2}$

(o) $-2x^{-3}-x^{-2}$ OR $\frac{-2}{x^3}-\frac{1}{x^2}$

(p) $(4x^2-1)(9x^2-4)$

$(4x^2-1)(18x)+(9x^2-4)(8x)$

$72x^3-18x+72x^3-32x$

$144x^3-50x$

(r) $(x^{99}+x^{98}+x^{97}+\dots+1)+(x-1)(99x^{98}+98x^{97}+97x^{96}+\dots)$

$(x^{99}+x^{98}+x^{97}+\dots+1)+(99x^{99}+98x^{98}+97x^{97}+\dots$

$-99x^{98}-98x^{97}-97x^{96}\dots$

$\cancel{x^{99}}+\cancel{x^{98}}+\cancel{x^{97}}+\dots x+1+99x^{99}+98x^{98}+97x^{97}+\dots+2x^2+x$

$-99x^{98}-98x^{97}-97x^{96}\dots-2x-1$

$100x^{99}+99x^{98}+98x^{97}+\dots+3x^2+2x+1$

$-99x^{98}-98x^{97}-\dots$

$\underline{100x^{99}}$

(s) $(-3x^2+1)(1-2x^3)(5x^2-x+1)$

$(-3x^2+6x^5+1-2x^3)(5x^2-x+1)$

$(-6x+30x^4-6x^2)(5x^2-x+1)+(10x-1)(-3x^2+6x^5+1-2x^3)$

$-30x^3+6x^2-6x+150x^6-30x^5+30x^4-30x^4+6x^3-6x^2-30x^3+60x^6+10x-20x^4+3x^2-6x^5-1+2x^3$

$210x^6-36x^5-20x^4-52x^3+3x^2+4x-1$

(t) $\frac{(x^2-4)(x^2+4)}{x^4}=\frac{vu'-uv'}{v^2}$ $\frac{x^4-16}{x^4}=\frac{x^4(4x^3)-(x^4-16)4x^3}{x^8}=\frac{4x^7-4x^7+64x^3}{x^8}=\frac{64}{x^5}$

$=64x^{-5}$

(u) $\frac{(2x+1)(3x+1)}{x^2+1}=\frac{6x^2+5x+1}{x^2+1}=\frac{(x^2+1)(12x+5)-(6x^2+5x+1)(2x)}{(x^2+1)^2}=\frac{12x^3+5x^2+12x+5-12x^3-10x^2-2x}{(x^2+1)^2}$

$=\frac{-5x^2+10x+5}{(x^2+1)^2}$

(4) $y=x(x+1)$

$y=3(3+1)=12$

$y=x(x+1)$

$y'=(x+1)+x=2x+1$

$y'=2(3)+1=7$

$y=7x+b$

$12=7(3)+b$

$\boxed{y=7x-9}$

$y=-\frac{1}{7}x+b$

$12=-\frac{1}{7}(3)+b$

$\boxed{y=-\frac{1}{7}x-28}$

(b) $y=x^3-x+1$

$y=2^3-2+1=7$

$y'=3x^2-1$

$y'=3(2)^2-1=11$

$y=11x+b$

$7=11(2)+b$

$\boxed{y=11x-15}$

$y=\frac{-1}{11}x+b$

$7=-\frac{1}{11}(2)+b$

$\frac{77}{-2}=b$

$\boxed{y=\frac{-1}{11}x-\frac{77}{2}}$