Calculus 12 Chain Rule Quiz Name: _____

Find the derivative.

1) $y=(3+x^3)^5$

2) $y=\frac{1}{4}(3x^2-2x+8)^{24}$

3) $y=\frac{1}{(3-4x)^2}$

4) $y=\sqrt{2-3x}$

5) $y=(4+5x)^2(8-x)^6$

6) $y=5x(x^2-4)^3$

7) $f(x) = (x-2)\sqrt{1-x^2}$

8) $f(x) = (x^2+1)^2\sqrt{x+1}$

9) $f(x) = \dfrac{x}{(x^2+1)^2}$

10) $y = \sin x \cos x$

11) $y = \tan 4x$

12) $f(x) = \dfrac{1-\cos x}{\sin x}$

Chain Rule Quiz key. /20

1. $y = (3+x^3)^5$

$y' = 5(3+x^3)^4 \cdot 3x^2$ (1)

$\boxed{y' = 15x^2(3+x^3)^4}$ ✓

2. $y = \frac{1}{4}(3x^2-2x+8)^{24}$

$y' = 6(3x^2-2x+8)^{23}(6x-2)$

$\boxed{y' = 6(6x-2)(3x^2-2x+8)^{23}}$ (1)

3. $y = \frac{1}{(3-4x)^2}$

$y = (3-4x)^{-2}$ ½

$y' = -2(3-4x)^{-3} \cdot -4$

$\boxed{y' = \frac{8}{(3-4x)^3}}$ (1)

4. $y = \sqrt{2-3x}$

$y = (2-3x)^{1/2}$ ½

$y' = \frac{1}{2}(2-3x)^{-1/2} \cdot -3$

$y' = \frac{-3}{2\sqrt{2-3x}}$ ✓ (1)

5. $y = (4+5x)^2(8-x)^6$

$u \quad v$

$y' = uv' + vu'$

(1) $= (4+5x)^2 6(8-x)^5(-1) + (8-x)^6 2(4+5x)^1 \cdot 5$

$= -6(4+5x)^2(8-x)^5 + 10(4+5x)(8-x)^6$ (1)

$= -2(4+5x)(8-x)^5[3(4+5x) - 5(8-x)]$ ½

$= -2(4+5x)(8-x)^5(12+15x-40+5x)$

$y' = -2(4+5x)(8-x)^5(20x-28)$

$y' = -8(4+5x)(8-x)^5(5x-7)$ ½

6. $y = 5x(x^2-4)^3$

$u \quad v$

$y' = uv' + vu'$

$= 5x \cdot 3(x^2-4)^2(2x) + (x^2-4)^3 \cdot 5$ (1)

$= 5(x^2-4)^2[6x^2 + (x^2-4)]$ ½

$= 5(x^2-4)^2(7x^2-4)$ ½

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$30x^2(x^2-4)^2 + 5(x^2-4)^3$ (1)

$5(x^2-4)(7x^2-4)$ (1)

7. $f(x) = (x-2)\sqrt{1-x^2}$

$f(x) = (x-2)(1-x^2)^{1/2}$ (u, v)

$= (x-2)\frac{1}{2}(1-x^2)^{-1/2} \cdot -2x + (1-x^2)^{1/2}$ (1)

$= (1-x^2)^{-1/2}\left[(x-2)(-x) + (1-x^2)\right]$

$= (1-x^2)^{-1/2}\left[-x^2+2x+1-x^2\right]$

$f'(x) = \frac{-2x^2+2x+1}{\sqrt{1-x^2}}$ (1)

$\frac{-x(x-2)}{\sqrt{1-x^2}}$

8. $f(x) = (x^2+1)^2\sqrt{x+1}$

$f(x) = (x^2+1)^2(x+1)^{1/2}$ (u, v)

$f'(x) = (x^2+1)^2 \cdot \frac{1}{2}(x+1)^{-1/2} + (x^2+1)' 4x (x+1)^{\frac{1}{2}}$ (1)

$f'(x) = \frac{1}{2}(x^2+1)(x+1)^{-1/2}\left[(x^2+1) + 8x(x+1)\right]$

$= \frac{1}{2}(x^2+1)(x+1)^{-1/2}\left[x^2+1+8x^2+8x\right]$ (1)

$= \frac{1}{2}(x^2+1)(x+1)^{-1/2}(9x^2+8x+1)$

$= \frac{9x}{2}\left(\frac{(x^2+1)(9x^2+8x+1)}{(x+1)2\sqrt{x+1}}\right)$

9. $f(x) = \frac{x}{(x^2+1)^2}$ (u, v)

$f'(x) = \frac{vu' - uv'}{v^2}$

$= \frac{(x^2+1)^2 - x(2(x^2+1)\cdot 2x}{(x^2+1)^4}$

$= \frac{(x^2+1)^2 - 4x^2(x^2+1)}{(x^2+1)^{4\,3}}$ (1)

$= \frac{(x^2+1) - 4x^2}{(x^2+1)^3}$

$f'x = \frac{-3x^2+1}{(x^2+1)^3}$ (1)

10. $y = \sin x \cos x$ (u, v)

$y' = uv' + vu'$

$= \sin x(-\sin x) + \cos x(\cos x)$

$= -\sin^2 x + \cos^2 x$ (1)

$= \cos 2x$ (1)

11. $y = \tan 4x$

$y' = \sec^2 4x \cdot 4$

$y' = 4\sec^2 4y$ ✓

12. $f(x) = \frac{1-\cos x}{\sin x}$ (u, v) $f'(x) = \frac{vu' - uv'}{v^2}$

$\frac{\sin x(\sin x) - (1-\cos x)(\cos x)}{(\sin x)^2}$ (1)

$= \frac{\sin^2 x - (\cos x - \cos^2 x)}{\sin^2 x}$

$= \frac{\sin^2 x + \cos^2 x - \cos x}{\sin^2 x}$ (1)

$f'(x) = \frac{1-\cos x}{\sin^2 x}$ (1)