

Name: _____

Date: _____

Practice
Derivative Quiz

2. Find the derivative of the following functions

a. $f(x) = 3\sqrt{x} - \sqrt[3]{x}$

b. $y = 2x + \frac{3}{\sqrt{x}}$

c. $f(x) = 3x^4 + 2x^2 + 6$

1. Let $f(x)$ be a function. Write the definition of the derivative of $f(x)$.

b. Using the definition in part (a), find $f'(x)$ for the function $f(x) = x^3 + 3$

3. Find the derivative of each of the following functions using the product rule.

a) $y = (x^2)(15x^3)$

b) $f(x) = (5x^7)\left(\frac{6}{x}\right)$

c) $y = (5\sqrt{x})(9x^3)$

4. Evaluate $\frac{d}{dx} \left[\frac{x^2 - 1}{x^2 + 1} \right]$

5. At what point is the tangent to the curve $f(x) = 4x^2 - x + 5$ parallel to the line $7x - y = 20$?

6. Let $f(x) = \begin{cases} 2x - x^2 & x \leq 0 \\ x + 1 & x \geq 1 \end{cases}$ if $0 < x < 1$

Where is f not differentiable?

7. Find the first and second derivative of $y = 2x^4 - 3x^2 + 1$.

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1. a. Let $f(x)$ be a function. Write the definition of the derivative of $f(x)$.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

b. Using the definition in part (a), find $f'(x)$ for the function $f(x) = x^3 + 3$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^3 + 3 - (x^3 + 3)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x^2 + 2xh + h^2)(x+h) - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 + x^2h + 2x^2h + 2xh^2 + xh^2 + h^3 - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} \\ &= \lim_{h \rightarrow 0} 3x^2 + 3xh + h^2 \\ &= 3x^2 \end{aligned}$$

2. Find the derivative of the following functions

a. $f(x) = 3\sqrt{x} - \sqrt[3]{x}$
 $= 3x^{1/2} - x^{1/3}$

$$\begin{aligned} f'(x) &= \frac{3}{2}x^{-1/2} - \frac{1}{3}x^{-2/3} \\ &= \frac{3}{2\sqrt{x}} - \frac{1}{3\sqrt[3]{x^2}} \end{aligned}$$

(3) $y = (x^2)^3 (5x^4)$

$$\begin{aligned} y &= (x^2)^3 (5x^4) + (5x^4)(3x^2) \\ y' &= 6x^2 + 45x^6 \\ &= 105x^6 \end{aligned}$$

(b) $f(x) = (5x^2)(6/x)$

$$\begin{aligned} f(x) &= (5x^2)(6x^{-1}) \\ &= (5x^2)(6x^{-1}) \\ &= (5x^2)(6x^{-2}) + (35x^0)(6x^{-1}) \\ &= -30x^5 + 210x^5 \\ f'(x) &= 180x^5 \end{aligned}$$

(c) $y = (5\sqrt{x^3})(9x^4)$

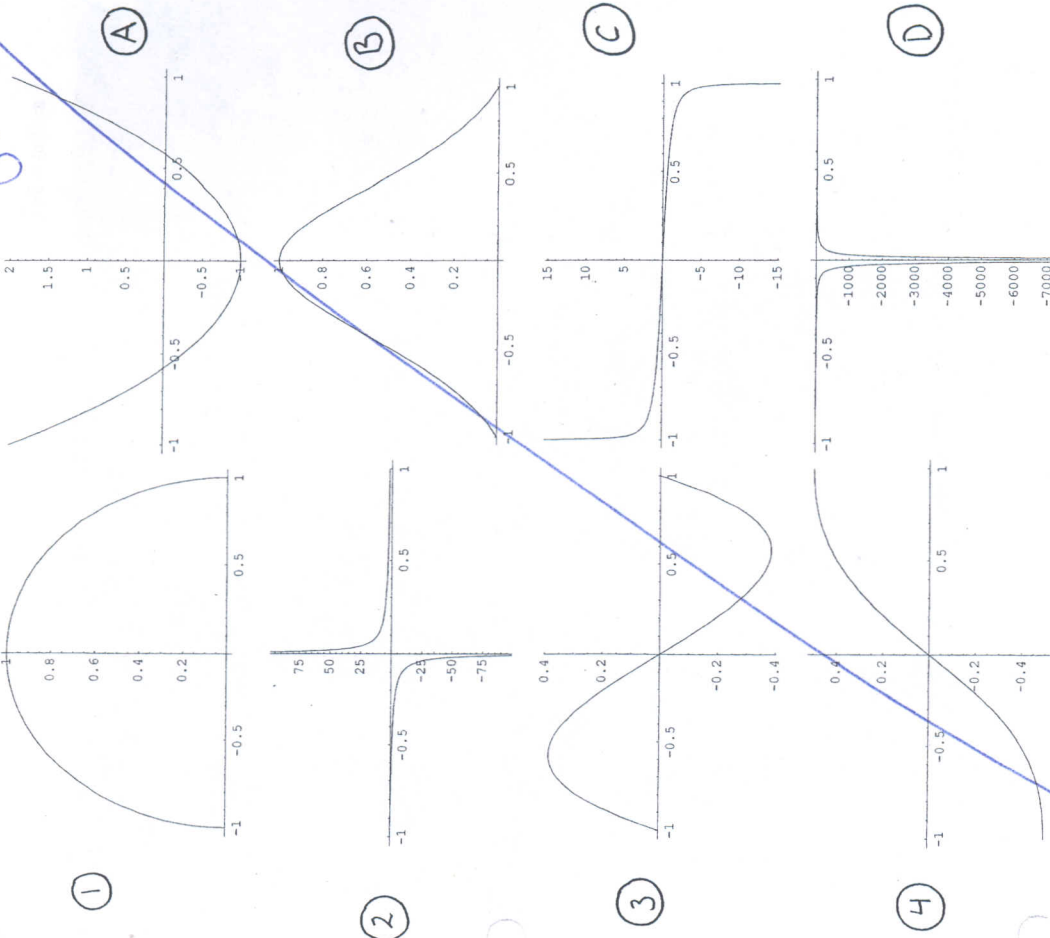
$$\begin{aligned} y &= (5x^{3/2})(9x^4) \\ y' &= \left(\frac{15}{2}x^{1/2}\right)(9x^4) + (36x^3)(5x^{3/2}) \\ &= \frac{135}{2}x^{9/2} + 180x^{9/2} \\ &= \frac{495}{2}x^{9/2} \text{ or } \frac{495\sqrt{x^9}}{2} \end{aligned}$$

c. $f(x) = 3x^4 + 2x^2 + 6$
 $f'(x) = 12x^3 + 4x$

(4) $f'(x) = \frac{(2x)(x^2+1) - (2x)(x^2-1)}{(x^2+1)^2}$

$$\begin{aligned} &= \frac{2x^3 + 2x - 2x^3 + 2x}{(x^2+1)^2} \\ &= \frac{4x}{(x^2+1)^2} \end{aligned}$$

3. Match the graph of the function $f(x)$ on the left with its derivative $f'(x)$ on the right



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5) 4. At what point is the tangent to the curve $f(x) = 4x^2 - x + 5$ parallel to the line $7x - y = 20$?

$$f'(x) = 8x - 1 = 7 \Rightarrow y = 7x - 20$$

$$8x = 8$$

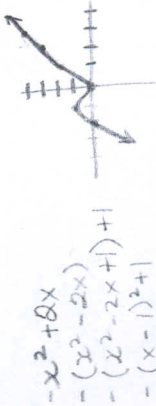
$$x = 1$$

$$y = 4(1)^2 - 1 + 5$$

$$= 4 - 1 + 5$$

$$= 8$$

point (1, 8)



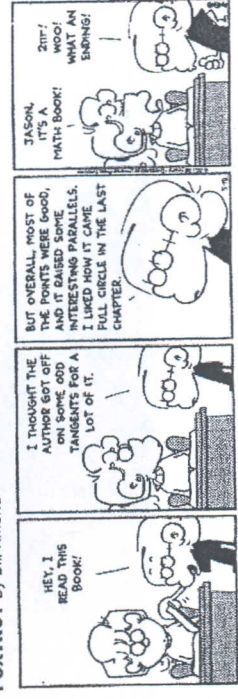
$x=1$ not differentiable?
 $f'(x) = 2 - 2x \quad x=0 \rightarrow 0$
 $f'(x) = 2 - 2 \rightarrow x=0 \rightarrow 0$
 $f'(x) = 1 \rightarrow$

$$y = 2x^4 - 3x^2 + 1$$

$$y' = 8x^3 - 6x$$

$$y'' = 24x^2 - 6$$

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1. — 2. — 3. — 4. —