Derivatives of Logarithmic Functions

1. Differentiate y with respect to x:

(a) $y = 6\ln x^3$

(b) $y = \ln(x^3 - 6)$

(c) $y = \ln\left(\frac{x^2}{6}\right)$

(d) $y = \frac{1}{2}\ln(5x^2 - 10x)$

2. Find $\frac{dy}{dx}$:

(a) $y = x\ln x - x + 8$

(b) $y = \ln\sqrt{4 - x^2}$

(c) $y = 4\ln^2(3x)$

(d) $y = \ln(x\sqrt{2 + 3x})$

(e) $y = \ln\left(\frac{x^2}{x^2 + 1}\right)$

3. Determine the derivative of the function $y = \ln(5x)$ from first principles.

4. Determine the derivative of the function $y = \ln(x^5)$ from first principles.

5. Differentiate y with respect to x and simplify:

(a) $y = \ln(x + \sqrt{x^2 + a^2})$

(b) $y = \ln\frac{x + a}{x - a}$

6. Differentiate y with respect to x and simplify:

(a) $y = \frac{1}{2}\sec^2 x + \ln\cos x$

(b) $y = \ln\csc^2 x$

(c) $y = \ln\tan^2(2x)$

(d) $y = \ln(\sec x + \tan x)$

(e) $y = \ln\sqrt{\frac{1 + \sin x}{1 - \sin x}}$

7. In each of the following, find d^2y/dx^2:

(a) $y = x^2\ln x$

(b) $y = x(\ln x)^2$

(c) $y = \frac{\ln x^2}{x}$

8. Find $\frac{dy}{dx}$ using logarithmic differentiation:

(a) $y = x^2\sqrt{x^2 - 4}$

(b) $y = x\sqrt{3x + 1}\sqrt{x - 5}$

(c) $y = (2x)^{1/2}\sqrt[3]{9x + 4}$

(d) $y = \frac{x^3\sqrt{3x - 7}}{\sqrt{6x + 2}}$

(e) $y = \sqrt{\frac{(x + 3)(x - 5)}{(x^2 - 4)(2x + 2)}}$

9. Differentiate the function

$$f(x) = \frac{x^2 - a^2}{x^2 + a^2}$$

using the quotient rule and using logarithmic differentiation. Show that the results are equivalent.

10. Determine an equation of the line tangent to each of the following functions at the specified point:

(a) $y = \ln x$,	$(1, 0)$
(b) $y = \ln x^3$,	$(1, 0)$
(c) $y = \ln(\cos x)$,	$(\pi/4, -\ln\sqrt{2})$
(d) $y = \sin(\ln x)$	$(1, 0)$

11. Find the angle at which the curves $y = \ln x$ and $y = \ln x^3$ intersect.

Exercise 12.2, Page 310

1. (a) $\frac{18}{x}$ (b) $\frac{3x^2}{x^3 - 6}$ (c) $\frac{2}{x}$ (d) $\frac{x - 1}{x^2 - 2x}$

2. (a) $\ln x$ (b) $\frac{-x}{4 - x^2}$ (c) $\frac{8\ln 3x}{x}$
(d) $\frac{4 + 9x}{2x(2 + 3x)}$ (e) $\frac{2}{x(x^2 + 1)}$

5. (a) $\frac{1}{\sqrt{x^2 + a^2}}$ (b) $\frac{-2a}{x^2 - a^2}$

6. (a) $\tan^3 x$ (b) $-2\cot x$ (c) $8\csc(4x)$
(d) $\sec x$ (e) $\sec x$

7. (a) $3 + 2\ln x$ (b) $\frac{2}{x}(1 + \ln x)$
(c) $\frac{1}{x^3}(-6 + 2\ln x)$

8. (a) $y\left[\frac{2}{x} + \frac{x}{x^2 - 4}\right]$
(b) $y\left[\frac{1}{x} + \frac{3}{2(3x + 1)} + \frac{1}{2(x - 5)}\right]$
(c) $y\left[\frac{1}{2x} + \frac{3}{9x + 4}\right]$
(d) $y\left[\frac{3}{x} + \frac{3}{2(3x - 7)} - \frac{3}{(6x + 2)}\right]$
(e) $\frac{y}{2}\left[\frac{1}{x + 3} + \frac{1}{x + 5} - \frac{2x}{x^2 - 4} - \frac{1}{x + 1}\right]$

9. $\frac{4a^2x}{(x^2 + a^2)^2}$

10. (a) $y = x - 1$ (b) $y = 3x - 3$
(c) $y = -x + \frac{\pi}{4} - \ln\sqrt{2}$ (d) $y = x - 1$

11. $\tan^{-1}\frac{1}{2}$ or 27°.

Derivatives of Exponential Functions

1. Find the derivative of the following functions with respect to x:

(a) $y = 5e^{-4x}$

(b) $y = \frac{1}{2}e^{x^2}$

(c) $y = x^4e^x$

(d) $y = \frac{e^{-x}}{x}$

(e) $y = \sqrt{1 + e^x}$

(f) $y = x + e^{\sqrt{x}}$

2. Find $\frac{dy}{dx}$:

(a) $y = 3^{3x}$

(b) $y = 10^{-x+2}$

(c) $y = 2^{-x} + x^{-2}$

(d) $y = 2^x \cdot x^2$

3. Determine an equation of the line tangent to the curve at the specified point:

(a) $y = e^{2x}$ $(1, e^2)$

(b) $y = \frac{e^{x^2}}{x}$ $(1, e)$

(c) $y = 2^{x^2}$ $(1, 2)$

(d) $y = e^{\sin x}$ $(\pi/6, \sqrt{e})$

4. In each of the following find d^2y/dx^2:

(a) $y = x^2e^{-x}$

(b) $y = 4xe^{x^2}$

(c) $y = e^{-x}\sin x$

(d) $y = \ln(xe^x)$

5. Show that if $y = e^x \cos 2x$, then

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 0$$

6. For what values of a does $x = Ce^{at}$ satisfy the differential equation

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} = 6x$$

7. Differentiate the following implicit functions and find y':

(a) $e^x + e^y = e^{x+y}$

(b) $ye^{2x} + xe^{2y} = 1$

8. Find the derivative of y with respect to x:

(a) $y = x^{\ln x}$

(b) $y = x^{\sqrt{x}}$

(c) $y = (6e^x)^{3x}$

(d) $y = (\sin x)^x$

9. Using logarithmic differentiation, show that the derivative of u^v is given by the formula

$$\frac{d}{dx}u^v = vu^{v-1} \cdot \frac{du}{dx} + u^v \ln u \cdot \frac{dv}{dx}$$

Note that this amounts to differentiating the function as if v were a constant and then as if u were a constant and adding the results.

Exercise 12.3, Page 314

1. (a) $-20e^{-4x}$ (b) xe^{x^2}
   (c) $4x^3e^x + x^4e^x$ (d) $-\frac{(x+1)e^{-x}}{x^2}$
   (e) $\frac{e^x}{2\sqrt{1+e^x}}$ (f) $1 + \frac{e^{\sqrt{x}}}{2\sqrt{x}}$
2. (a) $3 \cdot 3^{3x} \cdot \ln 3$ (b) $-10^{-x+2} \cdot \ln 10$
   (c) $-2^{-x} \cdot \ln 2 - 2x^{-3}$
   (d) $2^x \cdot 2x + x^2 \cdot 2^x \cdot \ln 2$
3. (a) $y = 2e^2x - e^2$ (b) $y = x$
   (c) $y = (4\ln 2)x + 2 - 4\ln 2$
   (d) $y = \frac{\sqrt{3e}}{2}x - \frac{\pi\sqrt{3e}}{12} + \sqrt{e}$
4. (a) $(x^2 - 4x + 2)e^{-x}$ (b) $(16x^3 + 24x)e^{x^2}$
   (c) $-2e^{-x}\cos x$ (d) $-\frac{1}{x^2}$
6. $-3, 2$
7. (a) $\frac{e^{x+y} - e^x}{e^y - e^{x+y}}$ (b) $-\frac{e^{2y} + 2ye^{2x}}{e^{2x} + 2xe^{2y}}$
8. (a) $\frac{2y}{x}\ln x$ (b) $y\left[\frac{1}{\sqrt{x}} + \frac{1}{2\sqrt{x}}\ln x\right]$
   (c) $y[6x + 3\ln 6]$ (d) $y[x\cot x + \ln(\sin x)]$