

Pre-Calculus 12 Practice Final 2
PART A: MULTIPLE CHOICE (non-calculator)
SECTION I

Answers are bolded for the multiple choice.

Value: 18 marks

Suggested Time: 40 minutes

1. Determine the exact value of $\csc \frac{5\pi}{3}$.

(a) $-\frac{\sqrt{3}}{2}$

(b) $\frac{1}{2}$

(c) 2

(d) $-\frac{2\sqrt{3}}{3}$

2. Which function has a period of 6π ?

(a) $f(x) = -\frac{1}{2} \sin 3x$

(b) $f(x) = \frac{1}{3} \sin 2x$

(c) $f(x) = \sin 6x$

(d) $f(x) = -\frac{1}{2} \sin \frac{x}{3}$

3. The terminal arm of angle θ in standard position intersects the unit circle at point (a, b) . Determine the value of $\sec \theta$.

(a) a

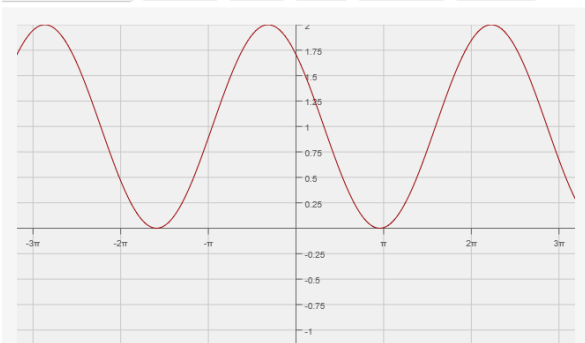
(b) b

(c) $\frac{1}{a}$

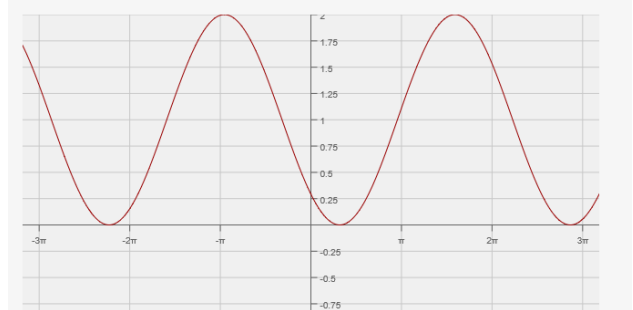
(d) $\frac{1}{b}$

4. Which graph represents the function $y = -\sin \frac{\pi}{4}(x-1) + 1$?

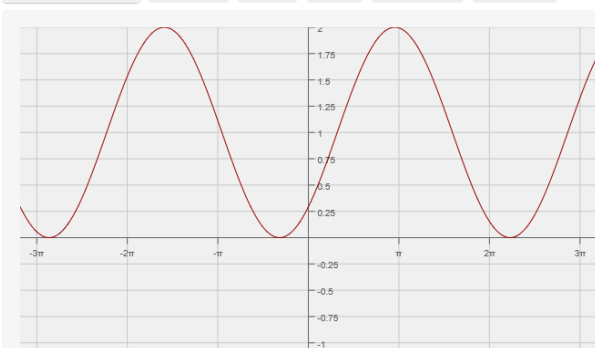
(a)



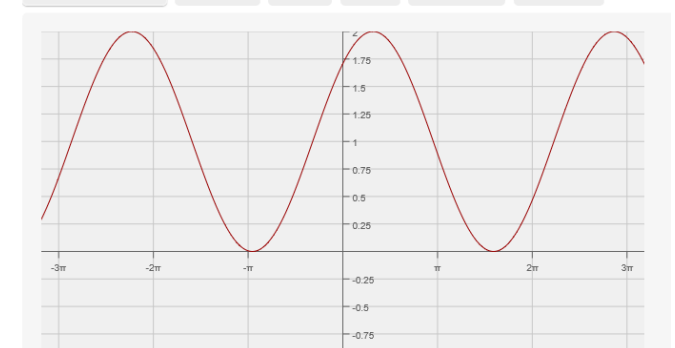
(c)



(b)



(d)



5. Which function has a range of $-6 \leq y \leq 2$?

- (a) $y = 4 \tan x - 2$ (b) $y = -4 \cos x + 2$ (c) $y = 4 \cos x - 2$ (d) $y = -4 \sin x + 2$

5. Simplify: $\cos(\pi - 2x)$

- (a) $-\cos 2x$ (b) $-\sin 2x$ (c) $\cos 2x$ (d) $\sin 2x$

6. Solve: $2 \cos^2 \theta - 3 \cos \theta + 1 = 0$ $0 \leq x < 2\pi$

- (a) $0, \frac{\pi}{6}, \frac{11\pi}{6}, 2\pi$ (b) $0, \frac{\pi}{3}, \frac{5\pi}{3}, 2\pi$ (c) $0, \frac{\pi}{3}, \frac{5\pi}{3}$ (d) $0, \frac{\pi}{6}, \frac{11\pi}{6}$

7. Determine the general solution of $\cos x + 2 \cos^2 x = 0$.

- (a) $x = n\pi, x = \frac{2\pi}{3} + 2n\pi, x = \frac{4\pi}{3} + 2n\pi$ (b) $x = \frac{\pi}{2} + n\pi, x = \frac{2\pi}{3} + 2n\pi, x = \frac{4\pi}{3} + 2n\pi$
(c) $x = n\pi, x = \frac{5\pi}{6} + 2n\pi, x = \frac{7\pi}{6} + 2n\pi$ (d) $x = \frac{\pi}{2} + n\pi, x = \frac{5\pi}{6} + 2n\pi, x = \frac{7\pi}{6} + 2n\pi$

8. Give the domain of the function $y = \log_3(2-x)$.

- (a) $x < 0$ (b) $x > 0$ (c) $x < 2$ (d) $x > 2$

9. Write $\log_7 x^2 + \log_7 x - \frac{5 \log_7 x}{2}$ as a single logarithm in simplest form.

- (a) $\frac{5}{2} \log_7 x^2$ (b) $\log_7(x^2 + x - x^{\frac{5}{2}})$ (c) $\log_7 \frac{(x^2)(x)}{x^{\frac{5}{2}}}$ (d) $\frac{1}{2} \log_7 x$

10. If $\log_2 7 = K$, then express $\log_2 \frac{\sqrt[5]{7}}{8}$ as an algebraic expression in terms of K .

- (a) $5k - 3$ (b) $\frac{1}{5}k - 3$ (c) $\frac{1}{5}k - 8$ (d) $5k - 8$

11. A John Deer Tractor was purchased for \$250 000. The tractor depreciates at 12% a year. Determine an expression for the value of the tractor t years after purchase.

- (a) $A = \frac{(0.88)^t}{250\,000}$ (b) $A = 250\,000(1.12)^t$
(c) $A = 250\,000(0.88)^t$ (d) $A = 250\,000(0.12)^t$

12. Solve for x: $\log(8x+4) = 1 + \log(x+1)$

- (a) 3 (b) $-\frac{3}{7}$ (c) -3 **(d) No solution**

13. Solve for x in terms of logarithms: $4^{2x-1} = 3^{x+2}$

- (a) $\frac{2 \log 3 + \log 4}{2 \log 4 + \log 3}$ (b) $\frac{2 \log 3 - \log 4}{2 \log 4 - \log 3}$ (c) $\frac{2 \log 3 + \log 4}{2 \log 4 - \log 3}$ (d) $\frac{2 \log 4 - \log 3}{2 \log 3 + \log 4}$

14. If the graph of the function $y = \sqrt{x}$ horizontally expanded by a factor of 3, vertically by a factor of 4 and then translated 2 units to the right, determine the equation of this new function.

- (a) $4y = \sqrt{3(x-2)}$ (b) $\frac{1}{4}y = \sqrt{\frac{1}{3}(x-2)}$ (c) $\frac{1}{4}y = \sqrt{3x-2}$ (d) $4y = \sqrt{\frac{1}{3}x-2}$

15. For this geometric sequence: $-5000, 500, -50, \dots$; which number below is the value of t_9 ?

- (a) 0.000 5 (b) -0.000 5 (c) 0.000 05 **(d) -0.000 05**

16. What is the sum of this infinite geometric series : $10 - \frac{20}{3} + \frac{40}{3} - \frac{80}{27} + \dots$?

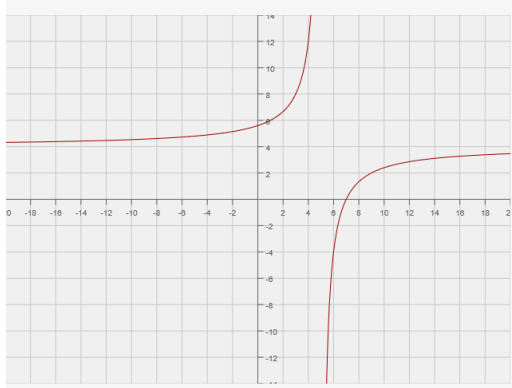
- (a) 30 (b) 4 (c) 6 (d) 20

17. The sum of the first 5 terms of a geometric series is $\frac{121}{81}$. The common ratio is $\frac{1}{3}$. What is the 2nd term?

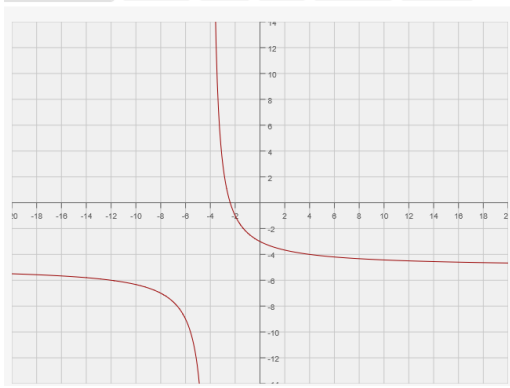
- (a) $\frac{1}{3}$ (b) 1 (c) $\frac{121}{3}$ **(d) $\frac{1}{3}$**

18. Which is the graph for the function $f(x) = \frac{-8}{x+4} - 5$?

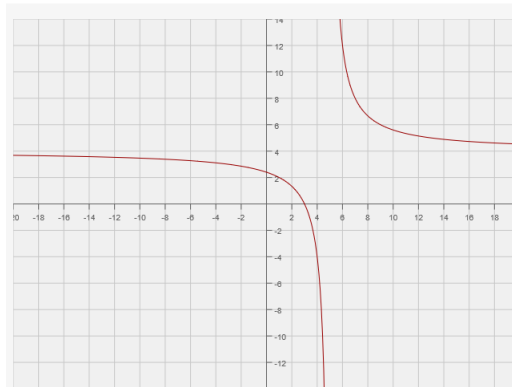
A.



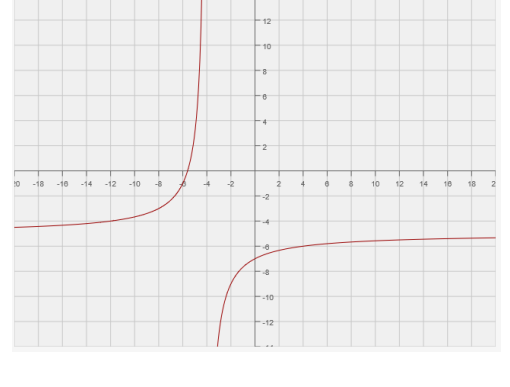
C.



B.



D.



PART B: MULTIPLE CHOICE (calculators permitted)
SECTION II

Value: 17 marks

Suggested Time: 45 minutes

19. What is the sum of the first 10 terms of this geometric series?

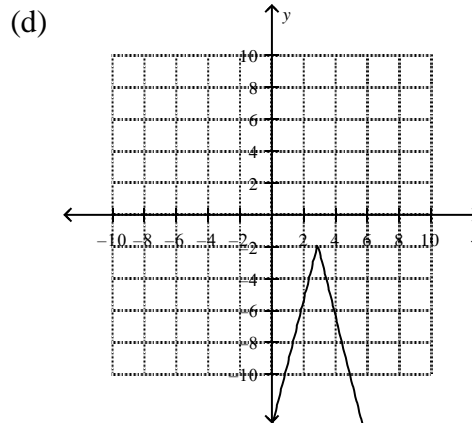
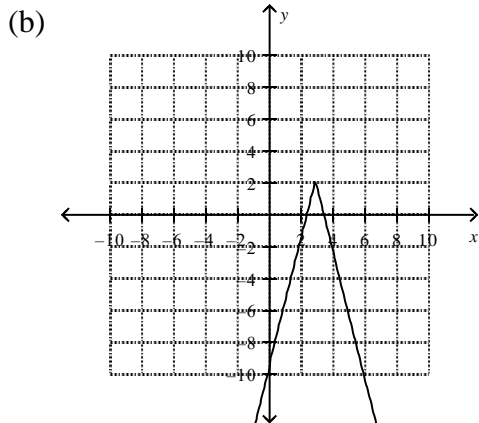
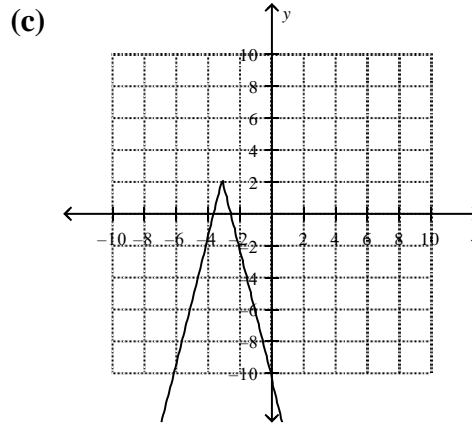
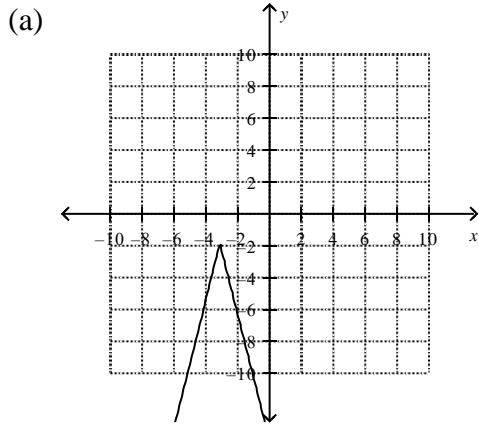
$$-12\,800 + 6\,400 - 3\,200 + 1\,600 - \dots$$

- (a) 8525 (b) -8525 (c) -8537.5 (d) 8537.5

20. The relation $x = \sqrt{9 - y^2}$ is compressed vertically by a factor of $\frac{1}{3}$, and then translated 1 unit to the right. Determine the equation of the transformed relation.

- (a) $x = \sqrt{9 - 9y^2} - 1$ (b) $x = \sqrt{9 - 9y^2} + 1$
 (c) $x = \sqrt{9 - \frac{y^2}{9}} + 1$ (d) $x = \sqrt{9 - \frac{y^2}{9}} - 1$

21. When the function $f(x) = |x|$ is transformed to $f(x) = -4|x + 3| + 2$, the graph looks like



22. Express as a single trigonometric functions: $4 - 8 \sin^2 6x$.

- A. $\cos 12x$
- B. $2 \cos 6x$
- C. $4 \cos 6x$
- D. $4 \cos 12x$**

23. Because of the action of the tides, water depth near the shore varies sinusoidally with time. At 3 AM the first maximum depth of 18m occurs. The first minimum depth of 4 m occurs $6 \frac{1}{2}$ hours later. Determine an equation for h metres at time t hours.

(a) $h = 7 \cos 2\pi \frac{(t-3)}{6.5} + 11$

(b) $h = 7 \cos 2\pi \frac{(t-3)}{13} + 11$

(c) $h = 11 \cos 2\pi \frac{(t-3)}{6.5} + 7$

(d) $h = 11 \cos 2\pi \frac{(t-3)}{13} + 7$

24. Determine the length (to 1 decimal place) of the arc that subtends an angle of 160° at the center of a circle with radius 12cm.

- (a) 148.0cm (b) 13.3 cm (c) 1920.0 cm (d) 33.5 cm

25. How many solutions are there for the equation $3 \sin 2x = -2$ in the interval $0 \leq x < 2\pi$?

- (a) 4 (b) 3 (c) 1 (d) 0

26. What value for k would make $x+2$ a factor of $y = x^3 + 10x^2 + 23x + k$?

- (a) $k=-1$ (b) $k=-14$ (c) $k=14$ (d) $k=1$

27. What are the coordinates of the invariant point(s) when the function $y = |x| - 2$ is reflected in the y-axis?

- (a) (2,-2) (b) (-2, 0) and (2, 0) (c) (0,-2) (d) (0,2)

28. Originally, there is 180 grams of an isotope, with a half-life of 7.2 days. How long will it be before 20 grams remain?

- (a) 22.8 days (b) 19.3 days (c) 25.6 days (d) 35.6 days

29. The pH of a solution is a measure of its acidity (0-7) or alkalinity (7-14) and is written as a logarithm of base 10. Neutral water has a pH of 7. Acid rain has a pH of 4.2. How many more times is acid compared to water

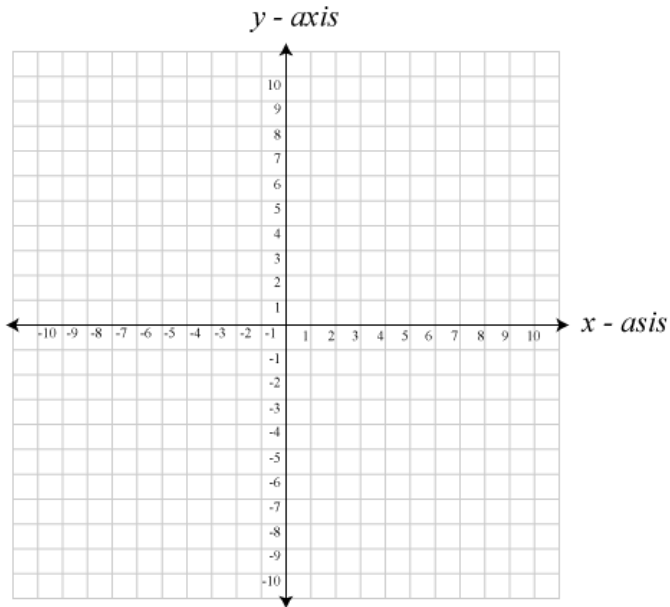
- (a) 0.6 (b) 2.8 (c) 1000 (d) 631

This is the end of the multiple choice section. Answer the remaining questions directly in the response booklet.

PART B: WRITTEN RESPONSE

Suggested Time: 30 minutes.

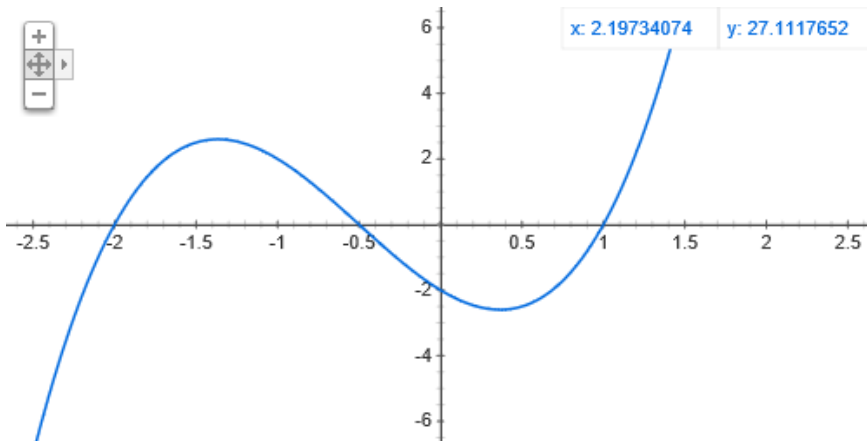
1. Graph $P(x) = 2x^3 + 3x^2 - 3x - 2$. Determine the x and y intercepts.



ANS:

x-intercepts: $(1,0)$, $(-1/2, 0)$, $(-2,0)$

y-intercept: $(0,-2)$



2. Algebraically determine the equation of the inverse of $f(x) = \frac{x-3}{x+7}$

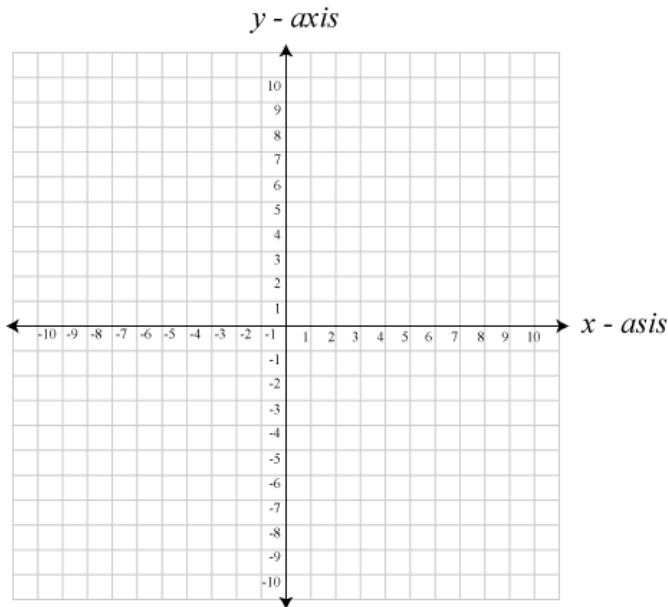
ANS: $f^{-1}(x) = \frac{-7x-3}{x-1}$

3. Solve algebraically:

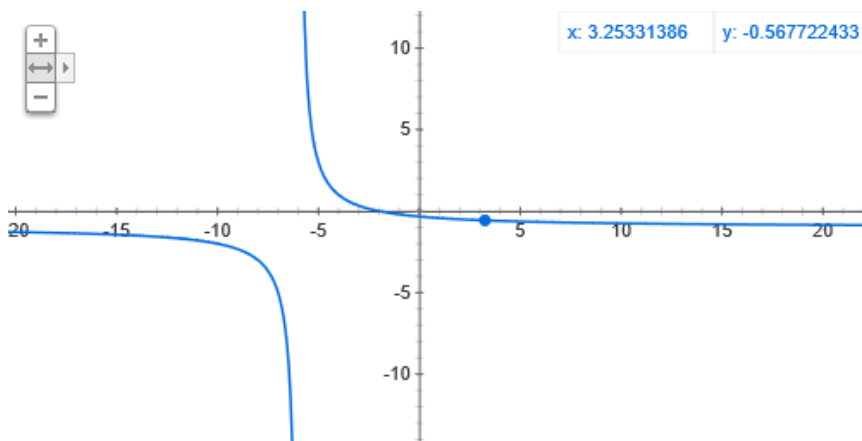
$$\log_2(x^2 - 2x)^7 = 21$$

ANS: $x=4,-2$

4. Write $y = \frac{-x-2}{x+6}$ in the form $y = \frac{a}{x-h} + k$. Use the equation to graph the function.



ANS: $\frac{4}{x+6} - 1$



5. Solve algebraically, where $\theta \in [-180^\circ, 180^\circ]$:

$$\sec^2 \theta - 2 \sec \theta - 3 = 0$$

If possible, answer using exact values; otherwise answer to at two decimal places of accuracy.

ANS:
 $180^\circ, \pm 70.53^\circ$

6. Determine the general solution for $\sin x = \cos 2x$. Solve algebraically over the set of real numbers using exact values.

$$\text{ANS: } \frac{\pi}{6} \pm \frac{4\pi n}{6}$$

Prove:

$$\frac{\sin 2\theta}{1 - \cos 2\theta} = \frac{\cos \theta + \cot \theta}{1 + \sin \theta}$$

ANS:

$$\frac{2 \sin \theta \cos \theta}{1 - \cos 2\theta}$$

$$\frac{\cos \theta + \frac{\cos \theta}{\sin \theta}}{1 + \sin \theta}$$

$$\frac{2 \sin \theta \cos \theta}{1 - (1 - 2 \sin^2 \theta)}$$

$$\frac{\frac{\cos \theta \sin \theta + \cos \theta}{\sin \theta}}{\frac{\sin \theta (1 + \sin \theta)}{\sin \theta}}$$

$$\frac{2 \sin \theta \cos \theta}{1 - 1 + 2 \sin^2 \theta}$$

$$\frac{\cos \theta (1 + \sin \theta)}{\sin \theta (\sin \theta + 1)}$$

$$\frac{\cos \theta}{\sin \theta}$$

$$\frac{\cos \theta}{\sin \theta}$$

$$\cot \theta$$

$$\cot \theta$$

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