Calculus 12: Review Part II

1. Use implicit differentiation to find $\frac{d^2y}{dx^2}$ if:

a) $4x^2 - 2y^2 = 9$

b) $x^2 + y^2 + 1 = 0$

2. Find the extrema: $f(x) = (x - 1)^2(x + 2)^2$

3. Find all the intervals for which the function is concave up and concave down.

a) $f(x) = x^2 - 4x + 3$

b) $f(x) = x^3 - 3x^2 + 1$

4. A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single – strand electric fence. With 800 m of wire at your disposal, what is the largest area you can enclose, and what are its dimensions?

5. The measurement of the radius of the end of a log is found to be 14 inches, with a possible error of ¼ inch. Use differentials to approximate the possible error in computing the area of the end of the log.

6. Air is being pumped into a spherical balloon at the rate of 4.5 cubic inches per minute. Find the rate of change of the radius when the radius is 2 inches.

7. Use a diagram and area formulas to evaluate the following integral. (Do not use the Fundamental Theorem of Calculus). (Exact answer)

$$\int_{-2}^{3} x - 1 \, dx$$

8. Use the Fundamental Theorem of Calculus to evaluate the following integrals.

(a) $\int_{0}^{1} x^3 + 3x^2 + 2 \, dx$

(b) $\int_{-1}^{3} \frac{x}{x^2+1} \, dx$

(c) $\int_{0}^{3} x e^{-x} \, dx$

(d) $\int_{-2}^{3} e^{2x} \cos 3x \, dx$

9. Solve the initial value problems:

$\frac{dy}{dx} = x^2 + x \quad y(2) = 1$ AND $\frac{d^2y}{dx^2} = 2 - 6x \quad y(0) = 1, y'(0) = 4$

10. Find the solution of the differential equation $dy/dx = ky$, that satisfies the given conditions.

(a) $k = -0.5$, $y(0) = 200$

(b) $y(0) = 60$, $y(10) = 30$

11. A national park is known to be capable of supporting no more than 150 grizzly bears, 30 bears are in the park at present. Find a logistic growth model $P(t)$ for the population with $k = 0.1$. Then find when the bear population will reach 75.

12. Find the area of the regions enclosed by the curves:

(a) $x + y^2 = 3$, $4x + y^2 = 0$

(b) $x + y^2 = 0$, $x + 3y^2 = 2$

Answer Key : Review Part II

① a) $4x^2 - 2y^2 = 9$

$$8x - 4y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{8x}{4y} = \frac{2x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{2x}{y}\right) = \frac{2y - 2x\, dy/dx}{y^2}$$

$$= \frac{2}{y} - \frac{2x}{y^2}\frac{dy}{dx}$$

$$= \frac{2}{y} - \frac{2x}{y^2}\left(\frac{2x}{y}\right)$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{2}{y} - \frac{4x^2}{y^3} \text{ when } y \neq 0}$$

b) $x^2 + y^2 + 1 = 0$

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{2x}{-2y} = \frac{-x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{-y + x\, dy/dx}{y^2}$$

$$= \frac{-1}{y} + \frac{x}{y^2}\left(-\frac{x}{y}\right)$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{-1}{y} - \frac{x^2}{y^3} \text{ when } y \neq 0}$$

$$\frac{-1}{y} - \frac{x^2}{y^3}$$

② $f(x) = (x-1)^2(x+2)^2$

$$f'(x) = 2(x-1)(x+2)^2 + 2(x+2)(x-1)^2 = 0$$

$$= 2(x-1)(x+2)[(x+2)+(x-1)] = 0 \quad \rightarrow 2x+1=0$$

$$x = 1, -2, -\tfrac{1}{2}$$

$f'<0$, $f'>0$, $f'<0$, $f'>0$ at -2, $-\frac{1}{2}$, 1

local maximum @ $x = -\frac{1}{2}$
minimum @ $x = -2, 1$

③ (a) $f(x) = x^2 - 4x + 3$

$f'(x) = 2x - 4$

$f''(x) = 2 > 0$

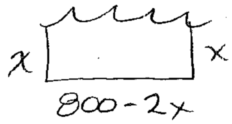
the curve $f(x)$ is concave up on $(-\infty, \infty)$

(b) $f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x$

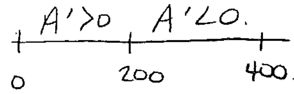
$f''(x) = 6x - 6$ is zero @ $x = 1$

Interval:	$x < 1$	$x > 1$
f'' :	$-$	$+$
f :	concave down	concave up

(4) 

$\text{Area} = x(800-2x) = 800x - 2x^2 = 0 \rightarrow 0 < x < 400$

$A' = 800 - 4x = 0 \rightarrow x = 200$



$x = 200 \rightarrow \max$

$A(200) = 800(200) - 2(200)^2 = \boxed{80\,000\ m^2}$

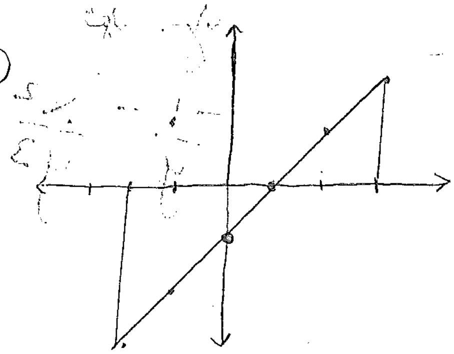
Dimensions: $\boxed{200\,m \text{ by } 400\,m}$

(5) $A = \pi r^2 \rightarrow \frac{dA}{dr} = 2\pi r\, dr = 2\pi(14)(0.25) = \boxed{7\pi}$

(6) $dV/dt = 4.5$ Find dr/dt when $r = 2$

$V = \frac{4}{3}\pi r^3 \rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \rightarrow \frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt}$

$= \frac{1}{4\pi(2)^2}(4.5) = \boxed{0.09\ in/min}$

(7) 

$A_1 = \frac{1}{2}bh = \frac{1}{2}(3)(3) = 9/2$

$A_2 = \frac{1}{2}bh = \frac{1}{2}(2)(2) = 2$

$A_{TOT} = 9/2 + 2 = \boxed{13/2}$

(8) $\int_0^1 x^3 + 3x^2 + 2\,dx = \frac{1}{4}x^4 + \cancel{x^3 + 2x}\Big|_0^1 = \frac{1}{4} + 1 + 2 = \boxed{13/4}$

(b) $\int_{-1}^3 \frac{x}{x^2+1} = \frac{1}{2}(\ln(x^2+1))\Big|_{-1}^3 = \frac{1}{2}\ln 10 - \frac{1}{2}\ln 2 = \boxed{\frac{1}{2}\ln 5 \approx 0.8047}$

(c) $\int_0^3 xe^{-x}dx = -xe^{-x} + \int e^{-x}dx = -xe^{-x} - e^{-x}\Big|_0^3 = \boxed{0.8009}$

(d) WINT $(e^{2x}\cos 3x, x, -2, 3) \approx \boxed{-18.186}$

(9) $\frac{dy}{dx} = x^2 + x \rightarrow \int dy = \int (x^2+x)dx \rightarrow y = \frac{1}{3}x^3 + \frac{1}{2}x^2 + c$

$1 = 8/3 + 2 + c \rightarrow c = -11/3$

$\boxed{y = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 11/3}$

(b) $y = x^2 - x^3 + 4x + 1$

(10) (a) $y(t) = 200e^{-0.5t}$

(b) $y(t) = 60e^{-(0.1\ln 2)t}$

OR $y(t) = 60 \cdot 2^{-t/10}$

(11) $P = \frac{150}{1+4e^{-0.1t}}$

(b) $t = \frac{\ln(0.25)}{-0.1}$

$= 13.86$ years

(12) (a) 8*

(b) $8/3$