Calculus 12
Implicit Differentiation
& Derivatives of Trig Functions
Quiz

NAME : _____

(A) Find the derivative dy/dx for each of the following.

1. $y^2 = 6x^2 + 3y^3$

2. $x^2y^4 = y^3 + 3x^2$

3. $(y^2 + 2)^3 = x^4y + 11$

4. (a) $2y + 5 - x^2 - y^3 = 0$
(b) Evaluate the derivative at the point $(2, -1)$

(B) Find the derivative of each of the following inverse trig. functions.

1. $y = \cos^{-1} 5x$

2. $y = \sin^{-1}(1-x^2)$

3. $y = 3\cos^{-1}(x^2+0.5)$

4. $y = \tan^{-1} 3x^4$

* BONUS: Find dy/dx if $x+y = \tan^{-1}(x^2+3y)$

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(A) Find the derivative dy/dx for each of the following.

1. $y^2 = 6x^2 + 3y^3$

$2y\frac{dy}{dx} = 12x + 9y^2\frac{dy}{dx}$

$2y\frac{dy}{dx} - 9y^2\frac{dy}{dx} = 12x$

$\frac{dy}{dx}(2y - 9y^2) = 12x$

$\frac{dy}{dx} = \frac{12x}{2y - 9y^2}$

2. $x^2y^4 = y^3 + 3x^2$

$2xy^4 + 4y^3\frac{dy}{dx}x^2 = 3y^2\frac{dy}{dx} + 6x$

$2xy^4 + 4x^2y^3\frac{dy}{dx} = 3y^2\frac{dy}{dx} + 6x$

$4x^2y^3\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 6x - 2xy^4$

$\frac{dy}{dx} = \frac{6x - 2xy^4}{4x^2y^3 - 3y^2}$

3. $(y^2 + 2)^3 = x^4y + 11$

$3(y^2+2)^2 \cdot 2y\frac{dy}{dx} = 4x^3y + \frac{dy}{dx}x^4$

$6y(y^2+2)^2\frac{dy}{dx} - x^4\frac{dy}{dx} = 4x^3y$

$\frac{dy}{dx} = \frac{4x^3y}{6y(y^2+2)^2 - x^4}$

$= \frac{4x^3y}{6y(y^4+4y^2+4) - x^4}$

$= \frac{4x^3y}{6y^5 + 24y^3 + 24y - x^4}$

4. (a) $2y + 5 - x^2 - y^3 = 0$

(b) Evaluate the derivative at the point $(2, -1)$

(a) $2\frac{dy}{dx} - 2x - 3y^2\frac{dy}{dx} = 0$

$(2 - 3y^2)\frac{dy}{dx} = 2x$

$\frac{dy}{dx} = \frac{2x}{2 - 3y^2}$

(b) $\frac{dy}{dx} = \frac{2(2)}{2 - 3(-1)^2}$

$= \frac{4}{-1} = -4$

(B) Find the derivative of each of the following inverse trig. functions.

1. $y = \cos^{-1} 5x$

$\frac{-1}{\sqrt{1-u^2}}$ · 5

$y' = \frac{-5}{\sqrt{1-25x^2}}$

2. $y = \sin^{-1}(1-x^2)$

$\frac{-1}{\sqrt{1-u^2}}$

$y' = \frac{-2x}{\sqrt{1-(1-x^2)^2}}$

$= \frac{-2x}{\sqrt{1-(1-2x^2+x^4)}} = \frac{-2x}{\sqrt{2x^2-x^4}}$

3. $y = 3\cos^{-1}(x^2+0.5)$

$\frac{-1}{\sqrt{1-u^2}}$ · $2x \cdot 3$

$= \frac{-6x}{\sqrt{1-(x^2+0.5)^2}}$

4. $y = \tan^{-1} 3x^4$

$\frac{1}{1+u^2}$

$\frac{12x^3}{1+(3x^4)^2}$

$\frac{12x^3}{1+9x^8}$

$= \frac{-2x}{\sqrt{x^2(2-x^2)}}$

$= \frac{-2x}{x\sqrt{2-x^2}}$

$= \frac{-2}{\sqrt{2-x^2}}$

* BONUS: Find dy/dx if $x+y = \tan^{-1}(x^2+3y)$

$1 + \frac{dy}{dx} = \frac{1}{1+(x^2+3y)^2} \cdot \left(2x + 3\frac{dy}{dx}\right)$

$1 + \frac{dy}{dx} = \frac{2x+3\frac{dy}{dx}}{1+(x^2+3y)^2} \Rightarrow 1 + \frac{dy}{dx} = \frac{2x}{1+(x^2+3y)^2} + \frac{3\frac{dy}{dx}}{1+(x^2+3y)^2}$

$\frac{dy}{dx} - \frac{3\frac{dy}{dx}}{1+(x^2+3y)^2} = \frac{2x}{1+(x^2+3y)^2} - 1$

$\frac{dy}{dx} = \frac{\frac{2x}{1+(x^2+3y)^2} - 1}{1 - \frac{3}{1+(x^2+3y)^2}} = \frac{2x-(1+(x^2+3y)^2)}{(1+(x^2+3y)^2-3} = \frac{2x-1-(x^2+3y)^2}{(x^2+3y)^2-2}$