

Integration Quiz

(1) Evaluate the indefinite integral:

$$\int (2+x^2)^2 dx$$

(2) Evaluate the following indefinite integrals:

(a) $\int \sec 4x \tan 4x dx$

(b) $\int \sin^5 3t \cos 3t dt$

(c) $\int \sqrt{2x-1} dx$

(d) $\int x(x^2+1)^3 dx$

(3) Solve the following initial-value problem:

$$f''(x) = x + \cos x \quad f(0) = 1, \quad f'(0) = 2$$

(4) Evaluate the following definite integral:

$$\int_{-1}^3 \frac{x}{x^2+1} dx$$

Integration Quiz

/15

(1) Evaluate the indefinite integral:

$$\begin{aligned} & \int (2+x^2)^2 dx \\ &= \int [(2+x^2)(2+x^2)] dx \\ &= \int (4+4x^2+x^4) dx \end{aligned}$$

$$\textcircled{1} \checkmark = 4x + \frac{4}{3}x^3 + \frac{1}{5}x^5 + c$$

$$\begin{aligned} u &= 2+x^2 \\ du &= 2x dx \end{aligned}$$

(2) Evaluate the following indefinite integrals:

$$\begin{aligned} \text{(a)} \int \sec 4x \tan 4x dx & \left| \begin{array}{l} \checkmark \textcircled{1} \\ u=4x \\ du=4 dx \\ dx = \frac{1}{4} du \end{array} \right. \\ \frac{1}{4} \int \sec u \tan u du & \left| \begin{array}{l} \checkmark \textcircled{1} \\ u=4x \\ du=4 dx \\ dx = \frac{1}{4} du \end{array} \right. \\ \frac{1}{4} \sec u + c & \\ \frac{1}{4} \sec 4x + c & \checkmark \end{aligned}$$

$$\begin{aligned} \text{(b)} \int \sin^5 3t \cos 3t dt & \checkmark \\ \frac{1}{3} \int u^5 du & \left| \begin{array}{l} \checkmark \textcircled{1} \\ u = \sin 3t \\ du = 3 \cos 3t dt \\ \cos 3t dt = \frac{1}{3} du \end{array} \right. \\ \frac{1}{3} \cdot \frac{1}{6} u^6 & \\ \frac{1}{18} \sin^6 3t + c & \checkmark \end{aligned}$$

$$\begin{aligned} \text{(c)} \int \sqrt{2x-1} dx & \checkmark \\ \frac{1}{2} \int u^{1/2} du & \left| \begin{array}{l} \checkmark \textcircled{1} \\ u = 2x-1 \\ du = 2 dx \\ \frac{1}{2} du = dx \end{array} \right. \\ \frac{1}{2} \cdot \frac{2}{3} u^{3/2} & \\ \frac{1}{3} (2x-1)^{3/2} + c & \checkmark \end{aligned}$$

$$\begin{aligned} \text{(d)} \int x(x^2+1)^3 dx & \checkmark \\ \frac{1}{2} \int u^3 du & \left| \begin{array}{l} \checkmark \textcircled{1} \\ u = x^2+1 \\ du = 2x dx \\ x dx = \frac{1}{2} du \end{array} \right. \\ \frac{1}{2} \cdot \frac{1}{4} u^4 & \\ \frac{1}{8} (x^2+1)^4 + c & \checkmark \end{aligned}$$

(3) Solve the following initial-value problem:

$$f''(x) = x + \cos x \quad f(0) = 1, \quad f'(0) = 2$$

$$f'(x) = \frac{1}{2}x^2 + \sin x + C \quad \checkmark \textcircled{1}$$

$$2 = C$$

$$f'(x) = \frac{1}{2}x^2 + \sin x + 2 \quad \checkmark .5$$

$$f(x) = \frac{1}{6}x^3 - \cos x + 2x + C \quad \checkmark \textcircled{1}$$

$$1 = 0 - 1 + 0 + C$$

$$C = 2$$

$$f(x) = \frac{1}{6}x^3 - \cos x + 2x + 2 \quad \checkmark .5$$

(4) Evaluate the following definite integral:

$$\int_{-1}^3 \frac{x}{x^2+1} dx$$

$$\frac{1}{2} \int_2^{10} \frac{1}{u} du \quad \checkmark .5$$

$$\left. \begin{array}{l} u = x^2 + 1 \quad \begin{array}{l} \nearrow 3^2+1=10 \\ \searrow (-1)^2+1=2 \end{array} \\ du = 2x dx \\ x dx = \frac{1}{2} du \end{array} \right| \textcircled{1}$$

$$\frac{1}{2} \ln u \Big|_2^{10} = \frac{1}{2} (\ln 10 - \ln 2) = 0.8047 \quad \checkmark .5$$

↑
preferred
answer.