Pre-Calculus 12
Note Package

Delview Secondary School

Notes to accompany the lesson for
the Pre-Calculus 12 Course
Prepared by Mrs. D. Couwenberghs
with credits to
Mrs. K. Samra
Mr. T. Vuroela
Mr. B. Outerbridge

Table of Contents

Pre-Calculus 12 Syllabus……………………………………………………………4
Unit 1 - Functions and Transformations……………………………………….6
Chapter 3 - Polynomial Functions……………………………………………...7
Chapter 2 & 9 – Radical & Rational Functions …….……………………………....26
Chapter 1 Transformations……………….…………………………………….........44
Unit 2 – Exponential and Log Functions…………………………………………75
Chapter 7&8 - Exponential and Log Functions ……………………………..……..76
Unit 3 – Trigonometry…………………………..…………………………………111
Chapter 4 – Trigonometry and the Unit Circle…………………………….…......112
Chapter 5 – Trigonometric Functions and Graphs…………………………..............131
Chapter 6 – Trigonometric Identities……………………………………………..152
Unit 4 – Rational Functions……………………………………………………...……177
Chapter 10 – Functions Operations…..……………………………………...…178
Chapter 11 – Permutations, Combinations and the Binomial Theorem……………..185

Delview Secondary School Department of Mathematics

Pre-Calculus 12 Syllabus

Course Rational

Learning to solve problems is the main reason for studying mathematics. In today's technological world, many people are finding that their job demands higher levels of mathematical ability than ever before.

Course Structure

The material for this course will be delivered in a variety of ways, the intention being to address the different learning style preferences of students enrolled in the course and at the same time, utilize suitably proven teaching methods to ensure successful concept development. It is expected that all students will endeavor to work equally well independently or collaboratively.

Textbook assignments will be given on a regular basis. On going student self-assessment will be an integral part of this course as students learn to take responsibility for their own learning.

Course Grade

Students will be assessed and evaluated on an individual basis using a variety of the following techniques:

Category	Description	Weight
Tests & Projects Teacher evaluation	• midterm testing • chapter tests • cumulative testing • projects	80 %
Quizzes Student self-assessment	• skill assessment • *may* have chance for rewrite	10 %
Assignments & Practice Teacher evaluation Student self-assessment & Concept and skill development	• review assignments • worksheets • group assignments • homework completion • in-class warm up questions	10 %

Work Habits Grade

Students are encouraged to demonstrate the kind of work habits that would be valued in any job situation. In general, qualities like punctuality, preparedness, perseverance through problem solving activities and diligence are minimum expectations in the work force. Students will be evaluated specifically as outlined in the Delview Student Agenda.

Materials Required For Class

Required Every Class	Required Occasionally
• 2 pencils (no erasable pens) • good eraser • Math textbook • 3-ring binder with loose-leaf paper • scientific calculator • ballpoint pen for marking work • 30 cm ruler (unbroken) • Math workbook • Math notebook	• graphing paper (always have some in binder) • pencil crayons • scissors • **graphing calculator** • miscellaneous minor supplies as required

Course Content

The Pre-Calculus 12 curriculum covers 9 main topics. It is meant to give the skills required for Calculus. For those students who wish to go on, the concepts will also be needed for college and university level mathematics.

Course Outline

Topic	Chapter Of Textbook
Polynomial Functions	3
Radical and Rational Functions	2&9
Transformations	1
Exponential and Logarithmic Functions	7&8
Trigonometry and the Unit Circle	4
Trigonometric Functions and Graphs	5
Trigonometric Identities	6
Function Operations	10
Permutations, Combinations and the Binomial Theorem	11

Unit 1
Functions

Chapter 3 - Polynomial Functions
Chapter 2&9 – Radical & Rational Functions
Chapter 1- Transformations of Functions

Polynomial Functions

The following are examples of *polynomial functions*. Give the degree, name of the polynomial and th leading term.

	Degree	Name	Leading Term
$f(x) = 3x - 6$	______	______________	__________
$f(x) = -4x^2 + 7x - 6$	______	______________	__________
$f(x) = x^3 - \frac{1}{5}x^2 + \frac{3}{7}x + \frac{4}{13}$	______	______________	__________
$f(x) = 3x^4 - \sqrt{2}\,x^3 + .7x^2 - x + 1059$	______	______________	__________
$f(x) = -\frac{4}{7}x^5 + x^4 - x^3 + 6x^2 - 3x + \sqrt{8}$	______	______________	__________

Definition

A polynomial function of degree n is any function of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_0$$

where n is a ________________________________ and $a_n, a_{n-1}, a_{n-2}, \ldots a_1, a_0$ are ________________________. a_n is called the leading coefficient.

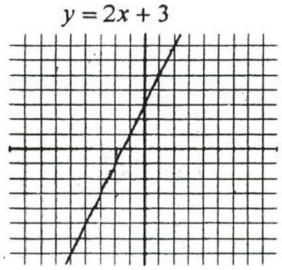
Why are the following functions **not** polynomial functions?

$f(x) = \sqrt{x} + 5$ $\qquad f(x) = \dfrac{x^2 + 3x - 2}{x + 2}$ $\qquad f(x) = 5^x + x^2$ $\qquad f(x) = \dfrac{2}{x} - 3$

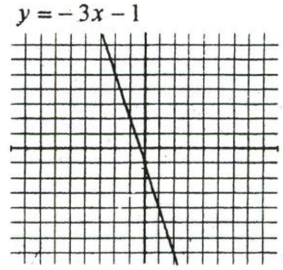
Graphs of polynomial functions. For each of the following state the degree of the function, the sign of the leading coefficient and the number or peaks/valleys in the graph. Also state any symmetry the graph shows.

Linear Functions

$y = 2x + 3$

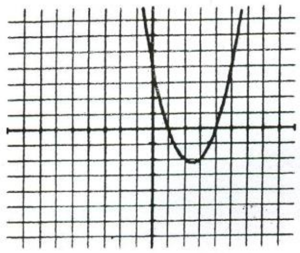


$y = -3x - 1$

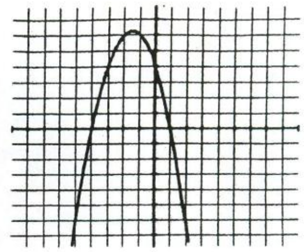


Quadratic Functions

$y = x^2 - 5x + 4$

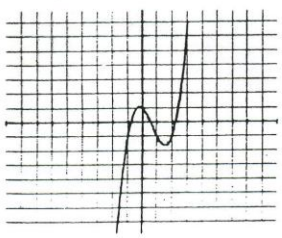


$y = -x^2 - 3x + 4$

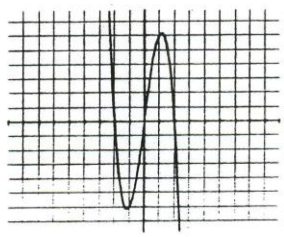


Cubic Functions

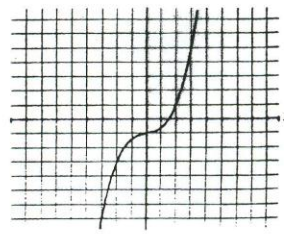
$y = x^3 - 2x^2 - x + 1$



$y = -2x^3 + 8x$

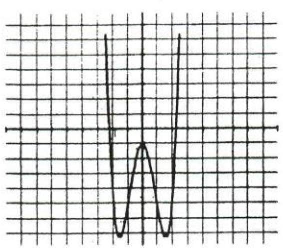


$y = .2x^3 + .2x - 1$

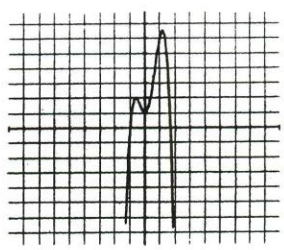


Quartic Functions

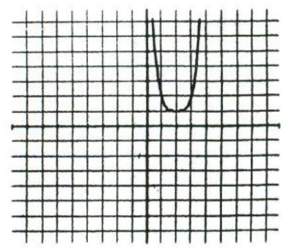
$y = x^4 - 5x^2 - 1$



$y = -4x^4 + 3x^3 + 6x^2 + 1$

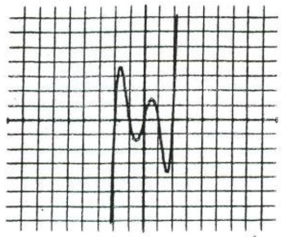


$y = x^4 - 8x^3 + 24x^2 - 32x + 17$

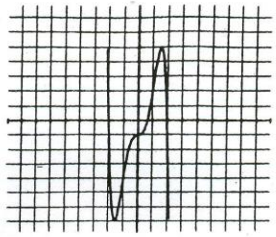


Quintic Functions

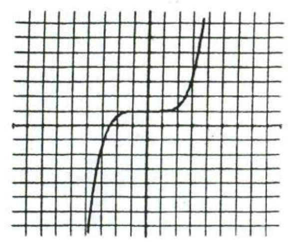
$y = x^5 - 5x^3 + 4x$



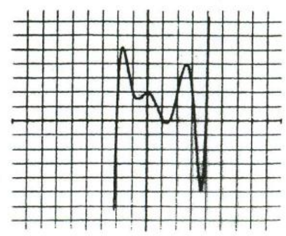
$y = -x^5 + 4x^3 - 1$



$y = .01x^5 + 1$



What might you expect the degree of the following function to be?



What does the degree of a polynomial tell you about the graph?

What does the leading coefficient tell you about the graph?

What does the constant term tell you about the graph?

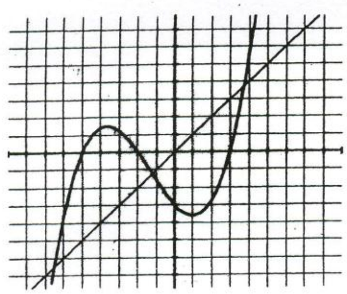
Predict what you expect the graph of $y = -2x^6 + 3x^5 - 7x^3 + x - 9$ to probably look like.

The Graph of a Polynomial Function

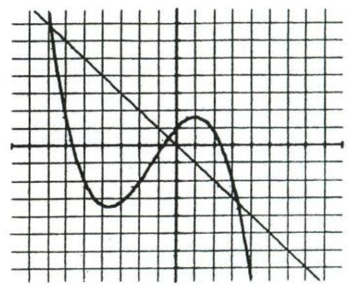
The graph of any polynomial function is a ______________, ______________ curve. The domain of all polynomial functions is ____________. The constant term is the ____________________. The maximum number of peaks and valleys is ______________________________________. The orientation of the graph is determined by the _________ of the ______________ coefficient.

Odd Degree

Positive leading coefficient

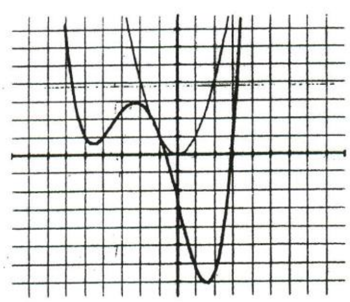


Negative leading coefficient

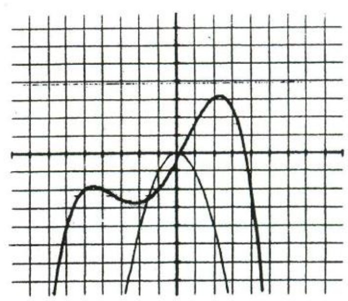


Even Degree

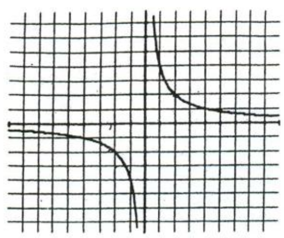
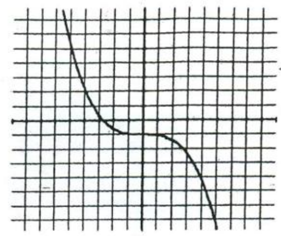
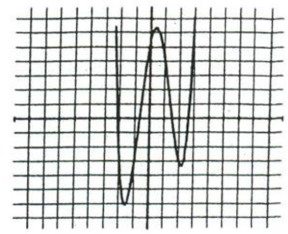
Positive leading coefficient

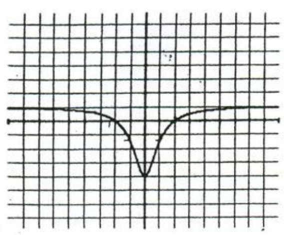
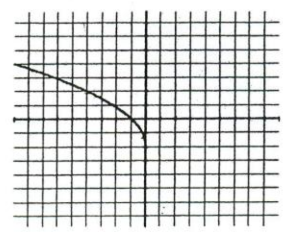
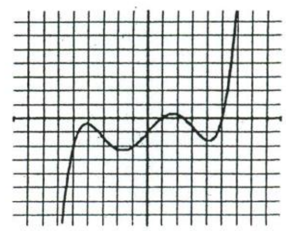


Negative leading coefficient

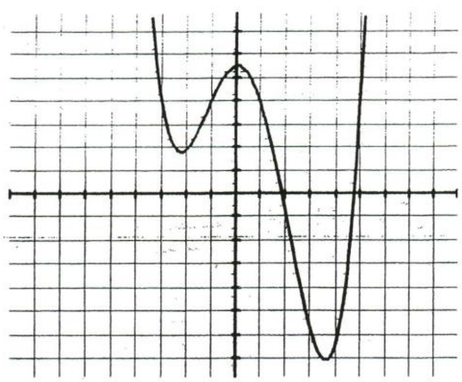


1. Which of the following could not be the graph of a polynomial function? Provide reasons.

2. Give the domain and range of the function graphed below. Graph the inverse of the function on the same grid. Is the inverse a function? Give the domain and range of the inverse.



Name: ________________

Date: ________________

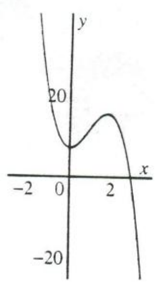
Warm-up 3.1

1. Determine if each function is a polynomial function or some other type of function.
   a) $f(x) = 2x^3 + x^2 - 5$
   b) $y = \dfrac{x^2 + x - 4}{x+2}$
   c) $f(x) = x(x-1)^2$

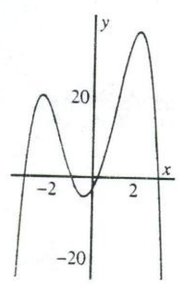
2. Identify the function that corresponds to each graph.

$f(x) = - x^3 - 3x^2 - 5x + 16$
$g(x) = - x^4 - 10x^2 - 5x + 5$
$h(x) = x^4 - x^3 + 11x + 9x - 3$
$g(x) = x^3 - x^2$

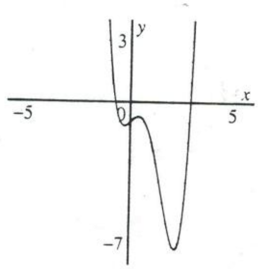
a)



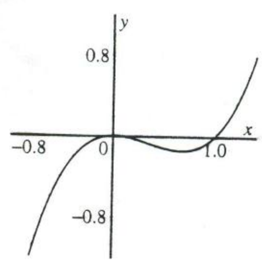
b)



c)



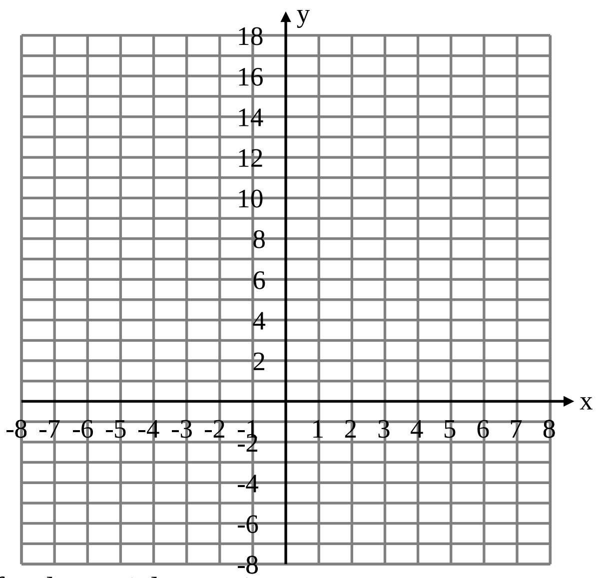
d)



Assignment: Pg. 72 #1-4, 6

3.3 Relating Polynomial Functions and Equations

$y = x^2 - 6x + 8$ $=$ Zeros: $x^2 - 6x + 8 = 0$ Roots:	$y = x^2 - 6x + 9$ $=$ Zeros: ______________________ $x^2 - 6x + 9 = 0$ Roots: ______________________	$y = x^2 - 6x + 10$ $=$ Zeros: $x^2 - 6x + 10 = 0$ Roots:

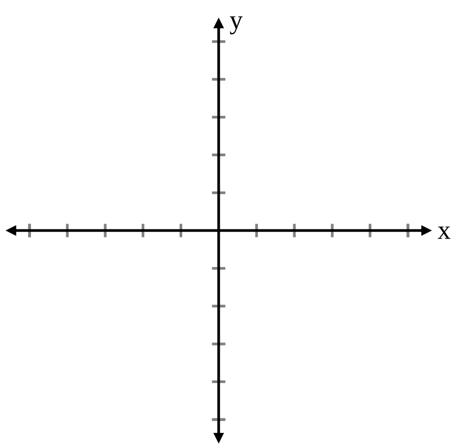


Examples: Roots and Factors of Polynomial Equations

<u>*Quadratic*</u>

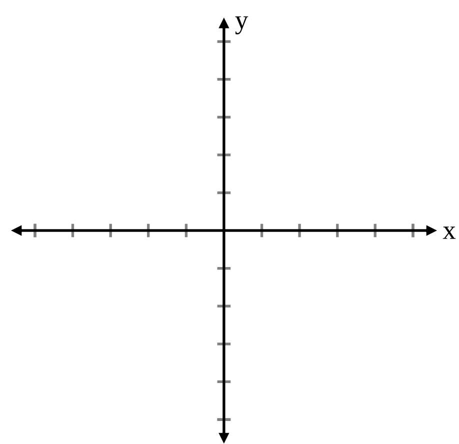
$f(x) = a(x + 2)(x - 3)$

Roots:



$f(x) = a(x + 2)^2$

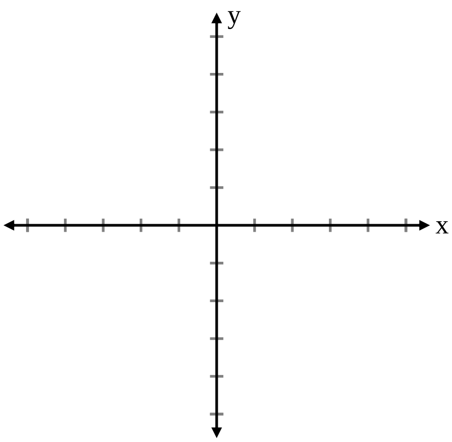
Roots:



Cubic

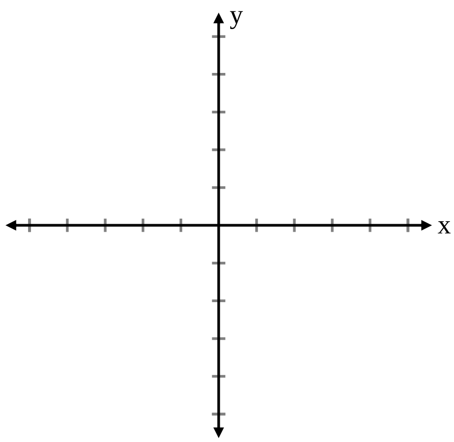
$f(x) = a(x + 2)(x - 1)(x - 3)$

Roots:



$f(x) = a(x + 2)(x - 3)^2$

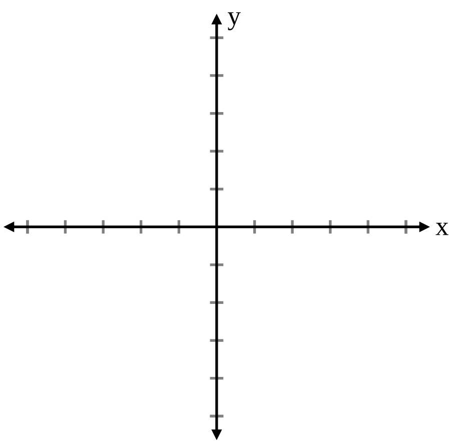
Roots:



Quartic

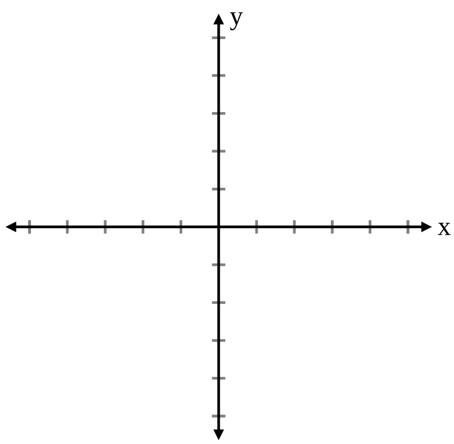
$f(x) = a(x + 2)(x - 1)(x - 3)^2$

Roots:



$f(x) = a(x + 2)(x - 3)^3$

Roots:



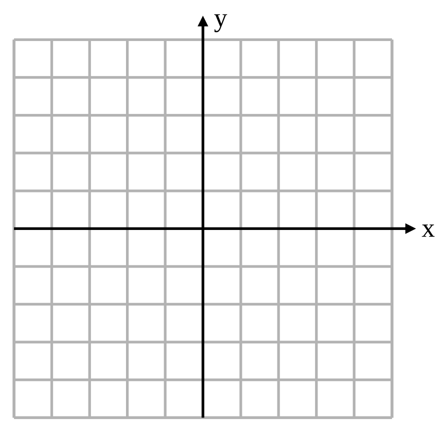
Examples:

(1) Write the equation of a quadratic function with zeros -5 and 12 which passes through (3, -5).

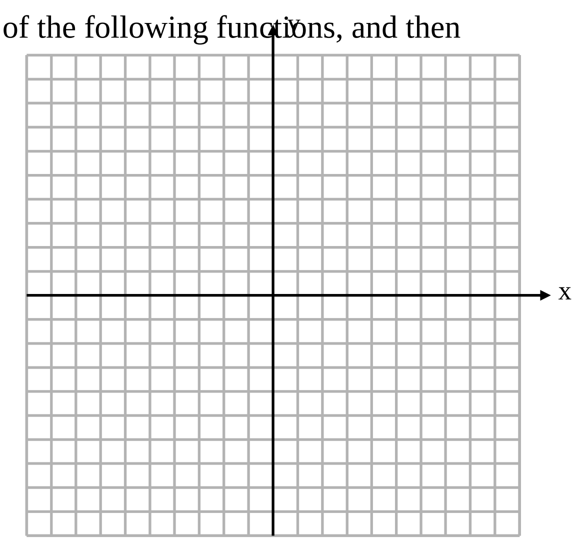
(2) Write the equation of a cubic function with zeros 2, 3, and -4 and which passes through (-1, 8).

(3) Write the equation of a quartic function with double zero at -3, single zeros of 2 and 8 and with
f(-2) = 20.

(4) Determine the equation of a quadratic function that has zeros -2 and 3, and whose graph passes through the point (1, 4). Graph the function.

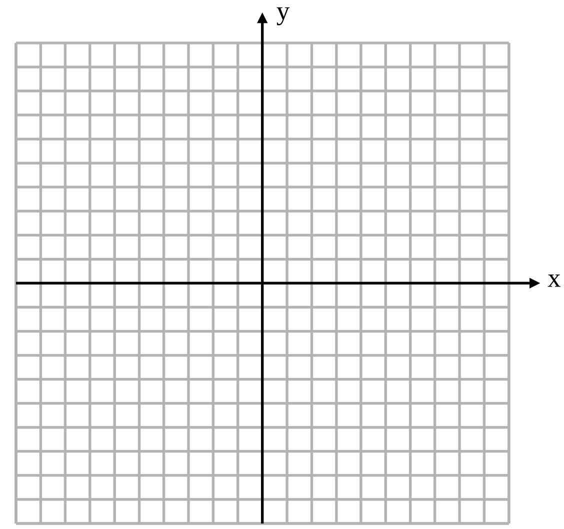


(5) Determine the zeros, leading term, and y – intercept of the following functions, and then use that information to sketch the function.

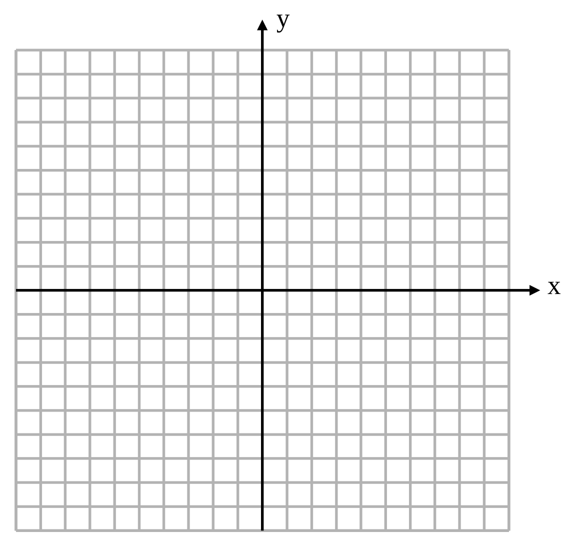


(a) y = (x – 3)(x + 2)(x – 5)(3x – 4)

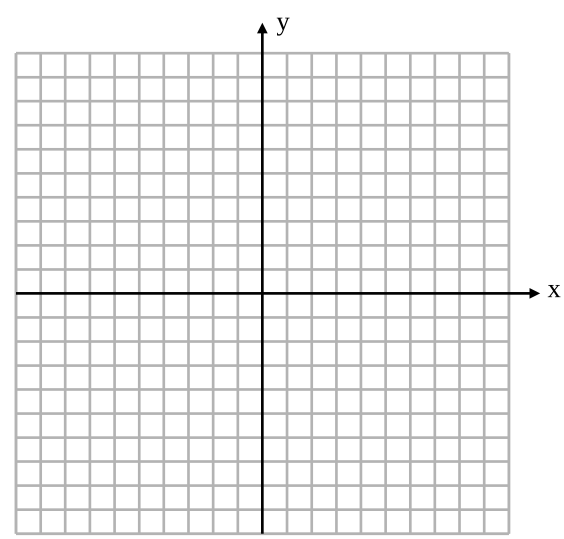
(b) $y = (x + 2)(x - 5)^2$



(c) $y = (x + 2)^2(x - 5)^2$



(d) $y = -2(4-x)(x - 1)(3 - 4x)$



(6) Determine the zeros of each function.

(a) $x^3 - 4x^2 - 12x = 0$

(b) $x^3 - x = 0$

(c) $x^3 - 3x^2 - 4x + 12 = 0$

(d) $3x^4 - 15x^2 + 12 = 0$

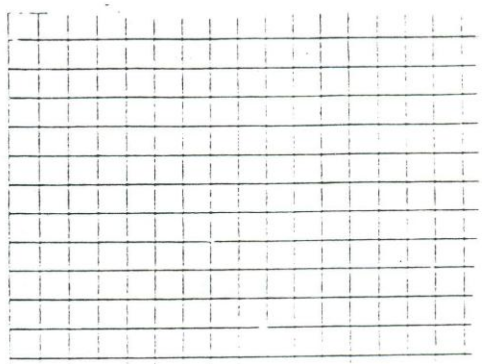
Assignment: *Pg. 97 1,2,5*

Name: ________________

Date: ________________

Warm-up 3.3

1. A cubic function with zeros −2,2,2; graph has y-intercept −16.
   a) Determine the equation of the function
   b) Sketch the function.

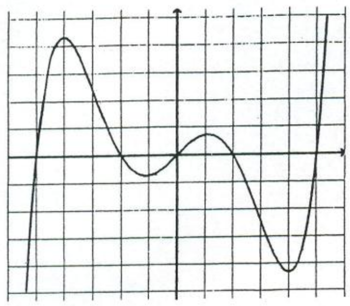


2. Determine the zeros of $f(x)=x^3-64x$

3. Determine the value of m in the equation $x^2+mx+25=0$ so that the roots are equal.

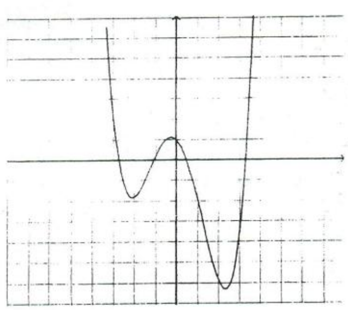
Math 11 Warmup

1. What could the degree of the following polynomial function be?



2. Determine the zeros of $y = 3x^3 - 8x^2 + 4x$ by factoring

3. Below is the graph of $y = .2x^4 - 2x^2 - x + 1$



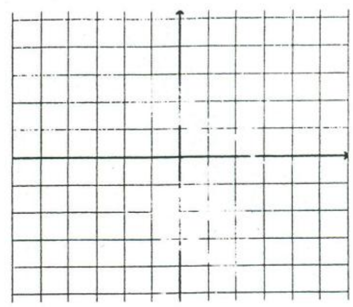
Give the domain and range of the function.

What is the equation of the inverse?

Graph the inverse on the grid above.

What are the domain and range of the inverse?

5. Sketch a polynomial function with degree 4, zeros of -2 and 3, double zero of 5 with a negative leading coefficient.



6. True or false

a) The degree of a polynomial indicates the number of roots the equation has.

b) The range of all polynomial functions is the Reals.

c) A polynomial equation with an odd degree has at least one root.

d) An even degree polynomial with a positive leading coefficient has an overall maximum.

e) The range of the inverse of any polynomial function is the Reals.

f) A quartic equation could have 3 roots.

g) A polynomial function with degree 12 could have 16 zeros.

<u>Division & The Remainder Theorem</u>

<u>Long Division:</u>
Use long division to divide the polynomial by the binomial. Remember restrictions.

$(x^3 + 3x^2 – 8x – 12) \div (x – 2)$ $\frac{P(x)}{x-a} = Q + \frac{R}{x-a}$

When we are trying to factor a polynomial, we would like to be able to determine whether a certain binomial divides evenly into it.

Example 1: Determine the remainder when the following polynomials are divided by the given binomial using long division. Remember to state restrictions.

(a) $(x^2 – 5x + 3) \div (x + 1)$

(b) $(2x^3 + 3x^2 – x + 7) \div (x – 2)$

(c) $(x^4 – 2x + 5) \div (x – 3)$

Synthetic Division:
Use synthetic division to divide the polynomial by the binomial:

$(x^3 + 3x^2 - 8x - 12) \div (x - 2)$ Dividend = Quotient × Divisor + Remainder

When we are trying to factor a polynomial, we would like to be able to determine whether a certain binomial divides evenly into it.
It would therefore be very helpful if we had a quick way of determining the remainder when we divide a polynomial by a binomial.

Example 1: Determine the remainder when the following polynomials are divided by the given binomial using synthetic division. Also evaluate the given polynomial at the indicated value.

(d) $(x^2 - 5x + 3) \div (x + 1)$	$p(x) = x^2 - 5x + 3$, evaluate $p(-1)$
(e) $(2x^3 + 3x^2 - x + 7) \div (x - 2)$	$p(x) = 2x^3 + 3x^2 - x + 7$, evaluate $p(2)$
(f) $(x^4 - 2x + 5) \div (x - 3)$	$p(x) = x^4 - 2x + 5$, evaluate $p(3)$

What is special about the indicated value when evaluating the polynomial?

What do you notice about synthetic division and evaluating the polynomial at the given value?

The Remainder Theorem
When the polynomial p(x) is divided by x – a the remainder is p(a).

This changes the problem of finding the remainder from being a division question to being a substitution question.

Example2: Determine the remainder when the following polynomials are divided by the given binomial.

(a) $(x^3 + 7x^2 - x + 5) \div (x + 2)$

(b) $(x^{10} + 5x - 3) \div (x - 2)$

Example 3: When $x^3 + kx^2 - 3x - 10$ is divided by $x + 2$ the remainder is 20. Determine the value of k.

Example 4: The remainder when $x^3 + 2x^2 - kx + 6$ is divided by $x - 3$ is -12. What is the remainder when the polynomial is divided by $x + 2$.

Assignment: Pg. 81 #1-7

FACTOR THEOREM

1. Factor: $f(x) = x^2 – 10x – 24$

If the roots are plugged back into the equation f(x), what will you get?

Therefore, ___________ and _____________ are factors of f(x).

2. Below is a table of values for $y = x^3 – 2x^2 – 5x + 6$. Using the table results only.

x	-4	-3	-2	-1	0	1	2	3	4
y	-70	-24	0	8	6	0	-4	0	18

a) What is the remainder when the polynomial is divided by x + 3?
b) Is x + 2 a factor of the polynomial?
c) How many factors of the polynomial does the table tell you?
d) What is the factored form of the polynomial?

The Factor Theorem

If p(a) = 0, then x – a is a factor of p(x).

Or

If a polynomial evaluated at a value k produces a value of 0, then x – k is a factor of the polynomial.

Example 1: Which of the following are factors of $x^3 + 6x^2 – x – 30$?

a) x + 1 (b) x + 3 (c) x – 1 (d) x - 2

What is the maximum number of factors that the polynomial above could have?

Once one factor is found, how can another factor be found?

We now have an easy way of determining whether a binomial is a factor of a given polynomial. The question now is: "Which binomials should we test?"

It is helpful to notice that $x^3 + 6x^2 – x + 10 = (x – 5)(x + 1)(x – 2)$

Factor Property

If x – k is a factor if a polynomial, then k must be a factor of the __________ ___________ of the polynomial. These are called integral zeros.

The factor property allows us to significantly narrow our search for **possible** factors.

Example 2: Find the integral zeros and factor completely: $x^3 + 8x^2 + 11x – 20$

Example 3: Find the integral zeros and factor completely: $21x^3 + 46x^2 + 21x - 4$

Assignment: Pg. 88 #2-9 + Pg. 100 #4
Quiz next class.

Warm-Up

1. Determine the roots exactly: $2x(x - 3) = 1$

2. Determine the roots exactly: $(5x + 1)(x + 2) = x$

3. What is the remainder when $x^3 - 5x^2 + 10x - 15$ is divided by $x - 3$?

4. Find the value of k such that when 2x3 + 9x2 + kx – 15 is divided by x + 5 the remainder is 0.

5. Divide the polynomial $3x^3 - 8x^2 + 3x + 2$ by $x - 2$ and factor the quotient.

Review Assignment due:_______________
Test:________________________________
More review: Pg. 103 #1-11