PRACTICE EXERCISE

1. For each of the following functions, state the intervals of increase and decrease, and determine the local maximum and minmum values.

   a) $y = 2x^2 - 16x + 39$

   b) $f(x) = \dfrac{2}{x-3}$

   c) $f(x) = 3\sin(x) + 2$, where $0 < x \leq 2\pi$

   d) $y = 2\cos^2(2x)$, where $0 < x < \dfrac{3\pi}{4}$

2. Find the concavity intervals and inflection points for each of the following functions.

   a) $f(x) = x^3 - 8x$

   b) $f(x) = 5x^3 - 5x^2 + 8$

   c) $f(x) = \dfrac{x^3 - 1}{x}$

   d) $f(x) = 5\cos(2x) - x$, where $0 < x < \pi$

2. Find the concavity intervals and inflection points for each of the following functions.

a) $f(x)=x^2-8x$

$f'(x)=2x-8$

$f''(x)=2$

Concave up $(-\infty,2)\cup(2,\infty)$

~~inflection point (2, 12)~~

No inflection point

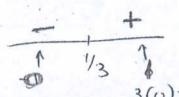
b) $f(x)=5x^3-5x^2+8$

$f'(x)=15x^2-10x$

$f''(x)=30x-10$

$30x-10=0$

$x=1/3$



$30(-1)-10$ $\quad$ $30(0)-10=-10$

Concave down $(-\infty,1/3)\cup$ Concave up $(1/3,\infty)$

Inflection point $(1/3, \frac{206}{27})$

c) $f(x)=\dfrac{x^2-1}{x}$

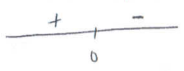
$f'(x)=\dfrac{2x^2-x^2+1}{x^2}$

$=\dfrac{x^2+1}{x^2}$

$f''(x)=\dfrac{2x^3-2x^3-2x}{x^4}$

$=-\dfrac{2x}{x^4}$

$=-\dfrac{2}{x^3}$



Concave up $(-\infty,0)$

Concave down $(0,\infty)$

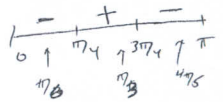
Inflection point $(0,$ undefined$)$

d) $f(x)=5\cos(2x)-x$, where $0<x<\pi$

$f'(x)=-10\sin 2x-1$

$f''(x)=-20\cos 2x$

$x=\pi/4,\ 3\pi/4$



Concave up $(\pi/4, 3\pi/4)$

Concave down $(0,\pi/4)\cup(3\pi/4,\pi)$

Inflection points

$(\pi/4, -\pi/4)$

$(3\pi/4, -\frac{3\pi}{4})$

PRACTICE EXERCISE

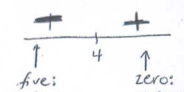
1. For each of the following functions, state the intervals of increase and decrease, and determine the local maximum and minmum values.

a) $y=2x^2-16x+39$

$y'=4x-16$

$4x-16=0$

$x=4$



Interval of increase:

$(-\infty,4)$

Interval of decrease:

$(4,\infty)$

minimum $x=4$

$y=2(4)^2-16(4)+39$

$=32-64+39$

$=7$

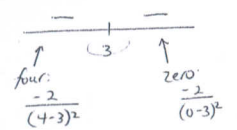
$(4,7)$

b) $f(x)=\dfrac{2}{x-3}$

$f'(x)=\dfrac{0-2}{(x-3)^2}=\dfrac{-2}{(x-3)^2}$

critical point: $(x-3)^2=0$

$x=3$.



Intervals of Decrease:

$(-\infty,3)\cup(3,\infty)$

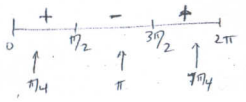
No MAX or MIN.

c) $f(x)=3\sin(x)+2$, where $0<x\leq 2\pi$

$f'(x)=3\cos x$

$\cos x=0$

$x=\pi/2,\ 3\pi/2$



Intervals of increase:

$(0,\pi/2)\cup(3\pi/2,2\pi]$

Interval of decrease:

$(\pi/2, 3\pi/2)$

Maximum: $(\pi/2, 5)$

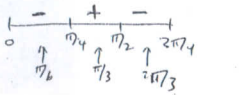
Minimum: $(3\pi/2, -1)$

d) $y=2\cos^2(2x)$, where $0<x<\dfrac{3\pi}{4}$

$y'=-4\cos(2x)\sin(2x)\cdot 2$

$=-8\cos 2x\sin 2x$

$x=\pi/4,\ \pi/2$



Intervals of increase:

$(\pi/4, \pi/2)$

Intervals of decrease

$(0,\pi/4)\cup(\pi/2, 3\pi/4)$

Maximum $(\pi/2, 2)$

Minimum $(\pi/4, 0)$