

QUIZ

2. Find the concavity intervals and inflection points for each of the following functions.

a) $f(x) = x^2 - x - 1$

b) $f(x) = 2x^4 - 4x^2 + 1$

1. For each of the following functions, state the intervals of increase and decrease, and determine the local maximum and minimum values.

a) $y = x^3 + x^2 - 8x + 5$

b) $f(x) = \frac{2}{x-3}$

c) $f(x) = \frac{x^2 - 1}{x}$

d) $f(x) = 5 \cos(2x) - x$, where $0 < x < \pi$

e) $f(x) = 3 \sin(x) + 2$, where $0 < x \leq 2\pi$

f) $y = 4 - \sqrt{x+2}$

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1. For each of the following functions, state the intervals of increase and decrease, and determine the local maximum and minimum values.

a) $y = x^3 + x^2 - 8x + 5$

$y' = 3x^2 + 2x - 8 = 0$ ✓
 $(x - 4)(x + 2) = 0$ ✓
 $x = 4, x = -2$

+	-	+	
$-\infty$	-2	4	∞

Increase $(-\infty, -2) \cup (4, \infty)$
 Decrease $(-2, 4)$
 Max $(-2, 17)$
 Min $(4, -41/2)$
/5

b) $f(x) = \frac{2}{x-3}$

$f'(x) = \frac{-2}{(x-3)^2}$ ✓
 $x = 3$ undefined ✓

-	-	
$-\infty$	3	∞

Decrease $(-\infty, 3) \cup (3, \infty)$
 or $(-\infty, \infty)$
 No max/min
/3

2. Find the concavity intervals and inflection points for each of the following functions.

a) $f(x) = x^2 - x - 1$

$f'(x) = 2x - 1$
 $f''(x) = 2$

Concave up: $(-\infty, 2) \cup (2, \infty)$
 or $(-\infty, \infty)$
 No inflection point
/3

b) $f(x) = 2x^4 - 4x^2 + 1$

$f'(x) = 8x^3 - 8x$
 $f''(x) = 24x^2 - 8$

$x^2 = 8/24 = 1/3$
 $x = \pm 1/\sqrt{3}$

+	-	+	
$-\infty$	$-1/\sqrt{3}$	$1/\sqrt{3}$	∞

Concave up: $(-\infty, -1/\sqrt{3}) \cup (1/\sqrt{3}, \infty)$
 Concave down: $(-1/\sqrt{3}, 1/\sqrt{3})$
 Inflection points: $(\pm 1/\sqrt{3}, -1/4)$
/4

d) $f(x) = 5 \cos(2x) - x$, where $0 < x < \pi$

$f'(x) = -10 \sin(2x) - 1$
 $f''(x) = -20 \cos(2x) - 2$

$x = \pi/4, 3\pi/4$

-	+	-	
0	$\pi/4$	$3\pi/4$	π

c) $f(x) = 3 \sin(x) + 2$, where $0 < x \leq 2\pi$

$f'(x) = 3 \cos x$ ✓
 $x = \pi/2, 3\pi/2$

+	-	+	
0	$\pi/2$	$3\pi/2$	2π

Increase $(0, \pi/2) \cup (3\pi/2, 2\pi)$ ✓
 Decrease $(\pi/2, 3\pi/2)$ ✓
 Max $(\pi/2, 5)$ ✓
 Min $(3\pi/2, -1)$ ✓
/5

d) $y = 4 - \sqrt{x+2}$

$y' = -1/2(x+2)^{-1/2}$
 $y'' = 1/4(x+2)^{-3/2}$

Decrease: $(-2, \infty)$ ✓
 No max/min ✓
/3

Concave up: $(-\infty, \infty)$
 Concave down: $(0, \infty)$
 Inflection points: $(0, \infty)$
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