Calculus Quiz
Related Rates

Name: ________
Mark: ___ /10

/5 (1) Sand falls from a conveyor belt at the rate of $10 m^3/min$ onto the top of a conical pile. The height of the pile is always three-eighths of the base diameter. When the pile is 4m high, how fast is the

(a) height changing?
(b) radius changing?

/4 (2) A spherical balloon is inflated with helium at the rate of $100\pi\ ft^3/min$.

(a) How fast is the balloon's radius increasing at the instant the radius is 5 ft?

(b) How fast is the surface area increasing at that instant?

/1 (3) If x, y, and z are lengths of the edges of a rectangular box, the common length of the box's diagonals is $s = \sqrt{x^2+y^2+z^2}$. How is ds/dt related to dx/dt, dy/dt and dz/dt?

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Name: Key

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/5 (1) Sand falls from a conveyor belt at the rate of $10m^3/min$ onto the top of a conical pile. The height of the pile is always three-eighths of the base diameter. When the pile is 4m high, how fast is the

(a) height changing?

(b) radius changing?

a) $V=\frac{1}{3}\pi r^2 h$ $\quad \frac{dV}{dt} = 10m^3/m$ $\quad d = 2r$

$h = 3/8 d$

$h = 3/4 r$

$\frac{4}{3}h = r$ ½

$V=\frac{1}{3}\pi(\frac{4}{3}h)^2 h$

½ $V=\frac{16}{27}\pi h^3$

½ $\frac{dV}{dt}=\frac{48}{27}\pi h^2\frac{dh}{dt}$

$=\frac{16}{9}\pi h^2\frac{dh}{dt}$

½ $10m^3/m = \frac{16}{9}\pi(4)^2\frac{dh}{dt}$

$\frac{10}{\frac{256\pi}{9}} = \frac{dh}{dt}$

$10 \cdot \frac{9}{256\pi} = \frac{dh}{dt}$

½ $\frac{90}{256\pi} = \frac{dh}{dt} = \frac{45}{128\pi}$ ≈ 0.11 m/min $= 11.2$ cm/min

½ The height is changing at a rate of $\frac{45}{128\pi}$ m/min

b) $\frac{4}{3}h = r$

$\frac{4}{3}\frac{dh}{dt} = \frac{dr}{dt}$ ½

$\frac{4}{3}\cdot\frac{45}{128\pi} = \frac{1}{2}$

$\frac{180}{384\pi} =$

$\frac{15}{32\pi} = \frac{dr}{dt}$ ½

½ The radius is changing at a rate of $\frac{15}{32\pi}$ m/min ≈ 0.15 m/m

/4 (2) A spherical balloon is inflated with helium at the rate of $100\pi\, ft^3/min$.

(a) How fast is the balloon's radius increasing at the instant the radius is 5 ft?

(b) How fast is the surface area increasing at that instant?

a) $V=\frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 100\pi ft^3/min$

½ $\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$

½ $100\pi = 4\pi(5)^2\frac{dr}{dt}$

½ $1 = \frac{dr}{dt}$

The radius is increasing @ 1 ft/min ½

b) $SA = 4\pi r^2$

$\frac{dSA}{dt} = 8\pi r\frac{dr}{dt}$ ½

$\frac{dSA}{dt} = 8\pi(5)(1)$ ½

$\frac{dSA}{dt} = 40\pi ft^2/min$ ½

½ The surface area is increasing at a rate of $40\pi ft^2/min$

/1 (3) If x, y, and z are lengths of the edges of a rectangular box, the common length of the box's diagonals is $s=\sqrt{x^2+y^2+z^2}$. How is ds/dt related to dx/dt, dy/dt and dz/dt?

$S=(x^2+y^2+z^2)^{1/2}$

$\frac{dS}{dt}=\frac{1}{2}(x^2+y^2+z^2)^{-1/2}\cdot 2x\frac{dx}{dt}+2y\frac{dy}{dt}+2z\frac{dz}{dt}$

$=\frac{\cancel{2}x\frac{dx}{dt}+\cancel{2}y\frac{dy}{dt}+\cancel{2}z\frac{dz}{dt}}{\cancel{2}\sqrt{x^2+y^2+z^2}} = \boxed{\frac{x\frac{dx}{dt}+y\frac{dy}{dt}+z\frac{dz}{dt}}{\sqrt{x^2+y^2+z^2}} = \frac{ds}{dt}}$ 1

$= (x\frac{dx}{dt}+y\frac{dy}{dt}+z\frac{dz}{dt}) \div s$